Combined Complexity of Probabilistic Query Evaluation

Mikaël Monet

October 12th, 2018

Relational Databases

- **Databases:** store information and query it later

Has_Specialty

doctor	specialty
Dr. Sneeze	allergologist
Dr. Bone	radiology
⋮	⋮

Appointment

patient	date	time	doctor
Nelly	17/04	11h	Dr. Sneeze
Jb	30/05	14h	Dr. Bone
Jb	05/11	15h	Dr. Sneeze
Jb	12/10	15h	Dr. Sneeze
⋮	⋮	⋮	⋮

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- **Query:** Retrieve patients having an appointment with a radiologist

→ SELECT patient FROM Appointment, Has_Specialty
WHERE Appointment.doctor = Has_Specialty.doctor
AND Has_Specialty.specialty = 'radiology'

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- **Query:** Retrieve patients having an appointment with a radiologist

$\rightarrow$ $p := \exists d'\, t\, d : \ \textbf{Appointment}(p, d', t, d) \wedge \textbf{Has\_Specialty}(d, \text{'radiology'})$

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→ SELECT doctor FROM Appointment

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- Applications: banks, institutions, libraries, movies, recipes, etc.

Uncertainty

- One usually assumes that the data is correct...

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- ... but in many cases it is not

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$\rightarrow$ Untrustworthy sources, automated information extraction, imprecise sensors in experimental sciences, etc.

Example: Optical Character Recognition

- Hospital wants to digitize and store all doctors' prescriptions

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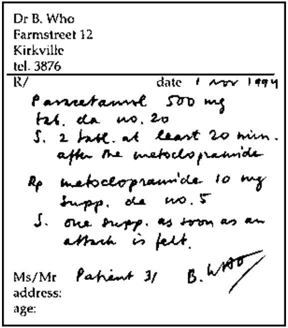
Dr B. Who
Farmstreet 12
Kirkville
tel. 3876

R/ date 1 nov 1994

Paracetamol 500 mg
tab. da no. 20
S. 2 tabl. at least 20 min.
after the metoclopramide

Rp metoclopramide 10 mg
supp. da no. 5
S. one supp. as soon as an
attack is felt.

Ms/Mr Patient 31 B. Who
address:
age:

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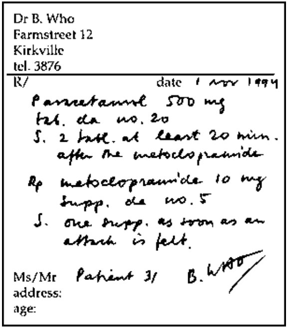
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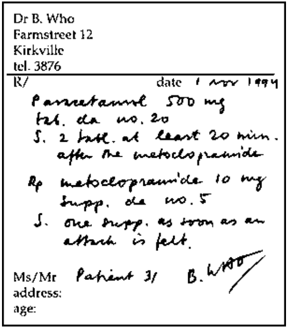
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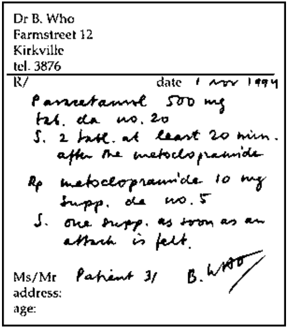
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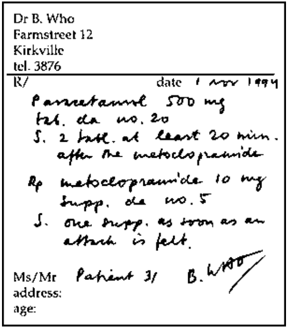
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person	ingredient
Billis	milk
Billis	shrimps
Bernard	eggs
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tiramisu
flapjacks
couscous
kougelhopf
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- But you can’t ask the candidate what the ingredients are (he might be too busy giving the presentation)
- What are the chances that you’ll be allergic to his tiramisu?

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$\rightarrow$ $\cdots$

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→ …

So maybe the tiramisu's ingredients will be:

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$\rightarrow$ sugar
$\rightarrow$ strawberries
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$\rightarrow$ $\dots$

So maybe the tiramisu's ingredients will be:

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- Real-world databases: more like 2^{1000} $\rightarrow$ **way too big**

Probabilistic Databases

- Need a framework to efficiently model this uncertainty and reason about it

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$\rightarrow$ Probabilistic Databases

- In this thesis: tuple independent databases (**TID**)

$\rightarrow$ Idea: assume independence across tuples

Tuple-independent databases (TID)

- Succinctly represent probabilistic data:
  - A relational database D
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$\rightarrow$ For $D' \subseteq D$, $\Pr(D') = (\prod_{F \in D'} \pi(F)) \times (\prod_{F \in D \setminus D'} (1 - \pi(F)))$

Example: TID

$D =$

Contains	
tiramisu	sugar
tiramisu	eggs

Example: TID

$D =$

C	
t	s
t	e

Example: TID

$(D, \pi) =$

C		
t	s	.5
t	e	.2

Example: TID

$(D, \pi) =$

C		
t	*s*	.5
t	*e*	.2

$.5 \times .2$

C	
t	*s*
t	*e*

Example: TID

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C		
t	*s*	.5
t	*e*	.2

$.5 \times .2$

C	
t	*s*
t	*e*

$.5 \times (1 - .2)$

C	
t	*s*

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C		
t	s	$.5$
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C	
t	s
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C	
t	s

$(1 - .5) \times .2$

C	
t	e

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C

Example: TID

$(D, \pi) =$

C		
t	s	$.5$
t	e	$.2$

$q = \exists x\, y\; C(x, y)$

$.5 \times .2$

C	
t	s
t	e

$.5 \times (1 - .2)$

C	
t	s

$(1 - .5) \times .2$

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$\Pr(D \models q) =$

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C	
t	s
t	e

✓

$.5 \times (1 - .2)$

C	
t	s

$(1 - .5) \times .2$

C	
t	e

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C	
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✓

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t	s

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C		
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C	
t	s
t	e

✓

$.5 \times (1 - .2)$

C	
t	s

✓

$(1 - .5) \times .2$

C	
t	e

$(1 - .5) \times (1 - .2)$

C

$$\Pr(D \models q) = .5 \times .2 + .5 \times (1 - .2)$$

Example: TID

$(D, \pi) =$

C		
t	s	.5
t	e	.2

$q = \exists x\, y\; C(x, y)$

$.5 \times .2$

C	
t	s
t	e

✔

$.5 \times (1 - .2)$

C	
t	s

✔

$(1 - .5) \times .2$

C	
t	e

✔

$(1 - .5) \times (1 - .2)$

C

$\Pr(D \models q) = .5 \times .2 + .5 \times (1 - .2)$

Example: TID

$$(D, \pi) = $$

C		
t	s	$.5$
t	e	$.2$

$$q = \exists x\, y\; C(x, y)$$

$.5 \times .2$		$.5 \times (1 - .2)$		$(1 - .5) \times .2$		$(1 - .5) \times (1 - .2)$
C		C		C		C
t	s	t	s			
t	e			t	e	
✔		✔		✔		

$$\Pr(D \models q) = .5 \times .2 + .5 \times (1 - .2) + (1 - .5) \times .2$$

Example: TID

$(D, \pi) =$

C		
t	s	.5
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$.5 \times .2$	
C	
t	s
t	e
✓	

$.5 \times (1 - .2)$	
C	
t	s
✓	

$(1 - .5) \times .2$	
C	
t	e
✓	

$(1 - .5) \times (1 - .2)$
C
✗

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✓

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✓

$(1 - .5) \times .2$

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t	*e*

✓

$(1 - .5) \times (1 - .2)$

C

✗

$$\Pr(D \models q) = .5 \times .2 + .5 \times (1 - .2) + (1 - .5) \times .2$$
$$= 1 - [(1 - .5) \times (1 - .2)]$$

Probabilistic query evaluation (PQE)

Let us fix:

- Class $\mathcal{D}$ of relational databases (e.g., acyclic, trees)
- Class $\mathcal{Q}$ of Boolean queries (e.g., paths, trees)

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Probabilistic query evaluation (PQE) problem for $\mathcal{Q}$ and $\mathcal{D}$:

- Given a query $q \in \mathcal{Q}$
- Given a database $D \in \mathcal{D}$ and a probability valuation π
- Compute the probability that (D, π) satisfies q

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- Given a database $D \in \mathcal{D}$ and a probability valuation π
- Compute the probability that (D, π) satisfies q

$\rightarrow$ $\Pr((D, \pi) \models q) = \sum_{D' \subseteq D,\ D' \models q} \Pr(D')$

Complexity of probabilistic query evaluation (PQE)

Question: what is the (data, combined) **complexity** of PQE depending on the class $\mathcal{D}$ of databases and class $\mathcal{Q}$ of queries?

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Wish list:

- PQE tractable in combined complexity
- **or** PQE tractable in the data, reasonable in the query

Data complexity results: related work (1/2)

- Existing data dichotomy result on queries [Dalvi & Suciu, 2012]
  - $\mathcal{D}_{\text{all}}$ = all possible databases
  - $\mathcal{Q} = \{q\}$, for q a UCQ (≈ SQL with **select**, **where**, **union**)

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→ q is 'safe' $\implies$ PQE is PTIME

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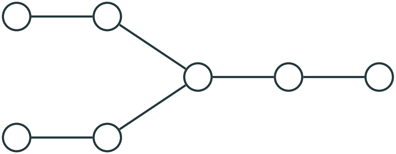
  → q is 'safe' $\Longrightarrow$ PQE is PTIME

  → q is not 'safe' $\Longrightarrow$ PQE is **#P-hard**
- Existing data dichotomy result on data

Treewidth by example:



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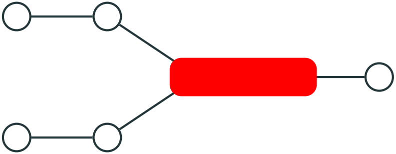

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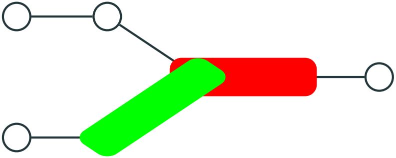

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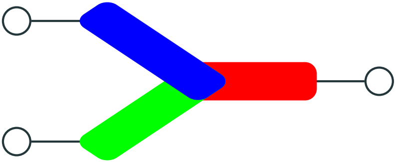

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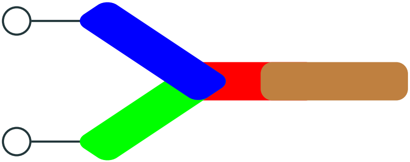

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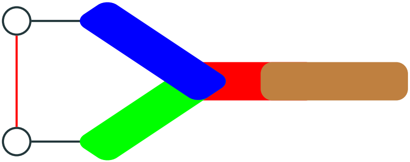

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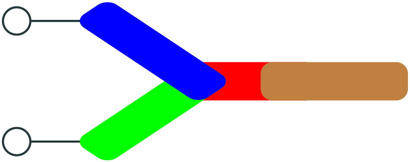

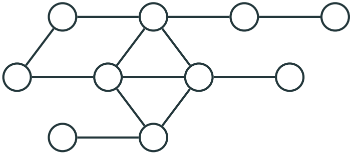

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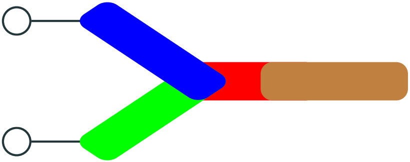

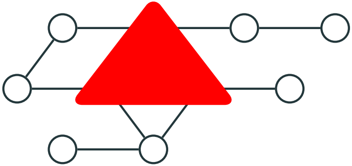

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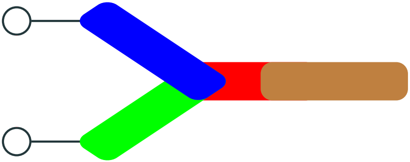

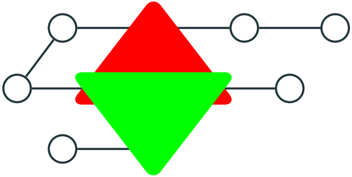

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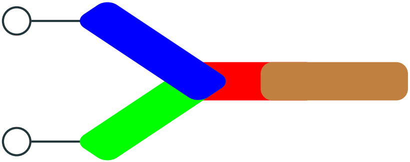

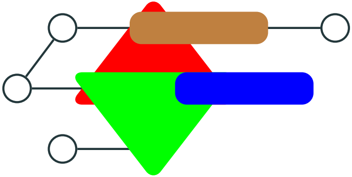

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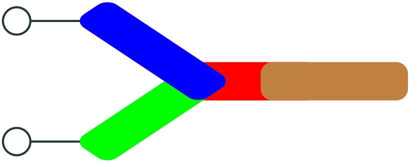

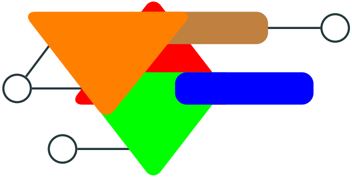

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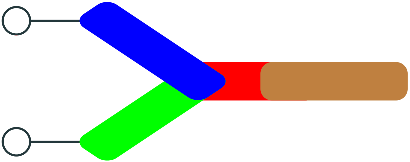

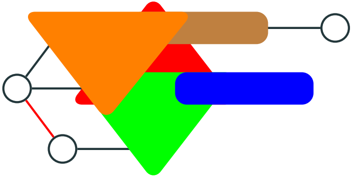

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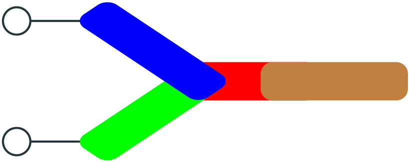

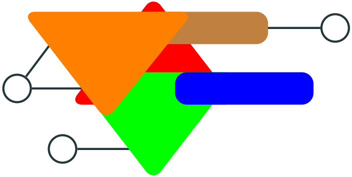

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$\rightarrow$ **Treelike**: the treewidth is **bounded by a constant**

Data complexity results: related work (2/2)

- Existing data dichotomy result on queries [Dalvi & Suciu, 2012]
  - $\mathcal{D}_{\text{all}}$ = all possible databases
  - $\mathcal{Q} = \{q\}$, for q a UCQ (≈ SQL with **select**, **where**, **union**)
  - → q is 'safe' $\implies$ PQE is PTIME
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What about combined complexity?

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During my thesis I have investigated:

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- Connections between safe queries and circuit classes from knowledge compilation
  → **AMW'2018** (with D. Olteanu)

PQE of conjunctive queries on binary signatures

Restrict to CQs on binary signatures

$\exists x\, y\, z\, t\; R(x, y) \wedge S(y, z) \wedge S(t, z)$

R		
b	c	.8
c	a	.1
c	d	.1

S		
a	b	1.
d	b	.05

Restrict to CQs on binary signatures

$\exists x\, y\, z\, t\; R(x,y) \wedge S(y,z) \wedge S(t,z) \quad \rightarrow \quad x \xrightarrow{R} y \xrightarrow{S} z \xleftarrow{S} t$

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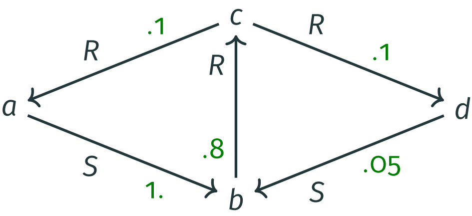

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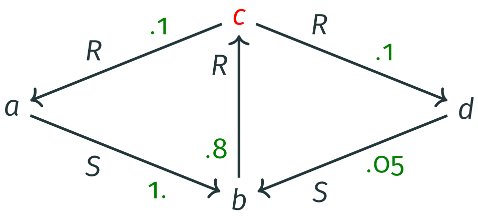

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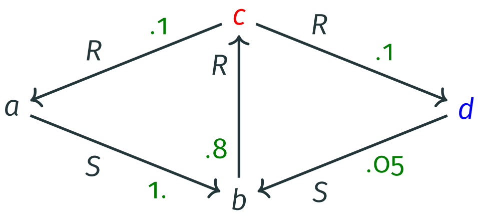

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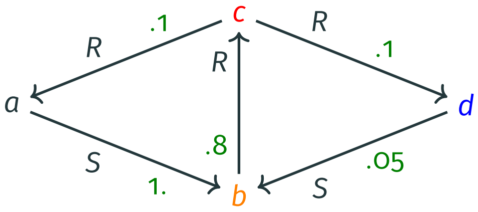

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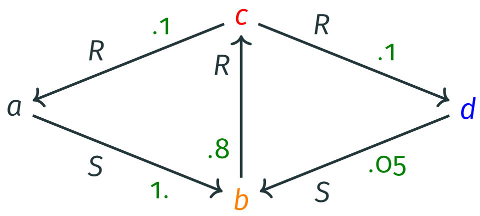

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$\mathcal{Q} =$ one-way paths (1WP), $\mathcal{D} =$ polytrees (PT)

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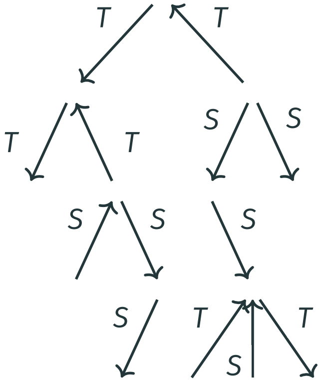

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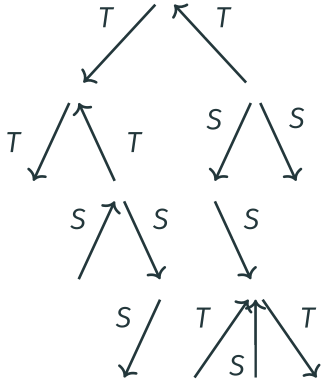

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Proposition

PQE of 1WP *on* PT *is* ***#P-hard***

$\mathcal{Q}$ = *one-way paths*, $\mathcal{D}$ = *polytrees*, without labels

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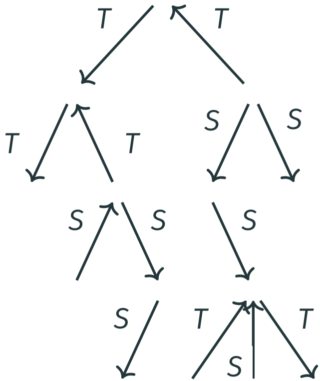

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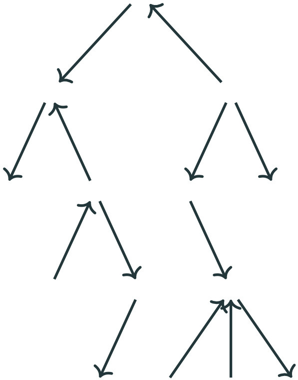

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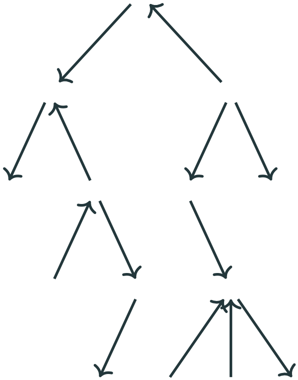

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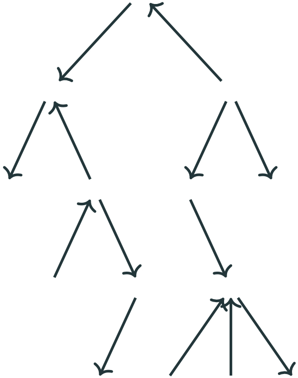

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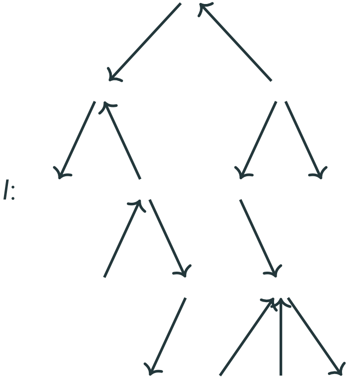

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- Labels have an impact!

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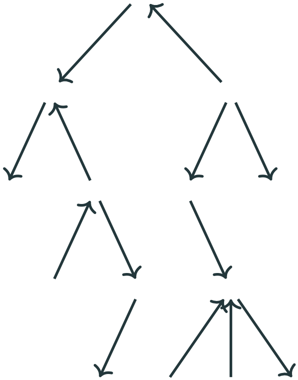

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Our graph classes

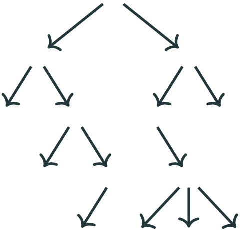
1WP

$\xrightarrow{R} \xrightarrow{S} \xrightarrow{S} \xrightarrow{T}$

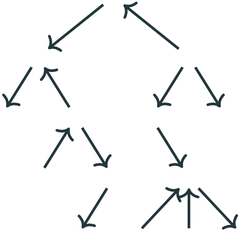
2WP

$\xrightarrow{R} \xleftarrow{S} \xleftarrow{S} \xrightarrow{T} \xleftarrow{R}$

DWT



PT



$$\text{1WP} \subseteq \text{2WP} \subseteq \text{PT}, \quad \text{1WP} \subseteq \text{DWT} \subseteq \text{PT}$$

$$\text{PT} \subseteq \text{Connected} \subseteq \text{All}$$

Results

$\downarrow Q$ $\quad D\rightarrow$	1WP	2WP	DWT	PT	Connected
1WP					
2WP					
DWT		PTIME			
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$\geqslant 2$ labels

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Summary

- Detailed study of the combined complexity of PQE
- Showed the importance of various features on the problem: labels, global orientation, branching, connectedness
- Established the complexity for all combinations of the graph classes we considered
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Drawbacks:

- Our graph classes may seem "arbitrary"
- Not yet a dichotomy, just starting to understand the problem
- Tractable cases very restricted

Lowering our expectations

What if we want the complexity to be:

- Tractable in the data
- **Not *too* horrible** in the **query**

Can we then support a **more expressive query language**?
(e.g., disjunctions, negations, recursion)

Non-probabilistic query evaluation on treelike databases

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The database class is parameterized

Idea:

- one parameter for the databases

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  → PQE for MSO on bounded-treewidth databases has linear data complexity [Amarilli, Bourhis, & Senellart, 2015]
  - Problem: nonelementary in the query $\left.2^{2^{\cdot^{\cdot^{\cdot^{|Q|}}}}}\right\}|Q|$

The database class is parameterized

Idea:

- one parameter for the databases
- and one parameter for the queries

Parameterized Complexity

Idea: one parameter k_D for the database (treewidth) AND one parameter k_Q for the query

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Definition

The problem is *fixed-parameter tractable (FPT) linear* if there exists a computable function f such that it can be solved in time $f(k_D, k_Q) \times |Q| \times |D|$

Results

1) A new language...

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- Given a CFG-Datalog program P with body-size k_P and a relational database D of treewidth k_D, checking if $D \models P$ can be done in time $f(k_P, k_D) \times |P| \times |D|$

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3) ... and also **FPT-linear** (combined) computation of provenance

- We design a new concise provenance representation based on cyclic Boolean circuits: ***cycluits***

Proof Sketch

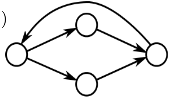
CFG-Datalog program P
of body-size ≤ k_P

1 C(x): Subway('Corvisart', x)
C(x): C(y) AND Subway(y, x)

2 Goal(): NOT C("Châtelet")

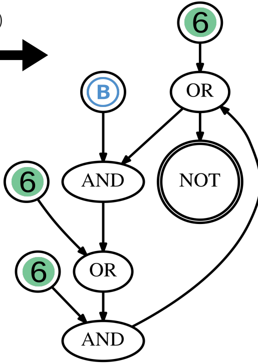
O(g′(k_P,k_D) |P|)

Two-way Alternating
Tree Automaton A



O(|A| |E|)

Provenance Cycluit



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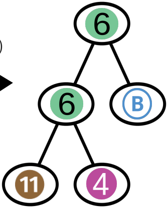

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Given a CFG-Datalog program P with body-size k_P and a relational database D of treewidth k_D, we can compute a cycluit representing the ***provenance*** *of P on D in time $f(k_P, k_D) \times |P| \times |D|$*

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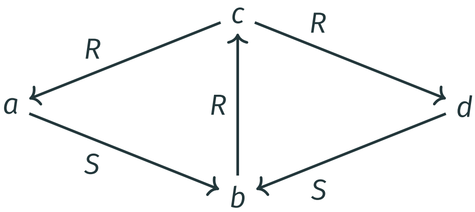
Can we lift this result to **probabilistic** evaluation?

Boolean cycluits to d-SDNNFs and lower bounds

Example: Provenance

$\exists x\, y\, z\; (R(x, y) \wedge S(y, z)) \vee (S(x, y) \wedge R(y, z))$



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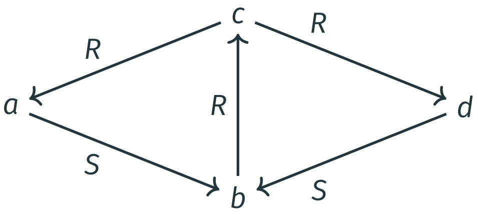
$\exists x\, y\, z\ (R(x,y) \land S(y,z)) \lor (S(x,y) \land R(y,z))$



$$\text{Prov}(q,D) = [S(a,b) \land (R(b,c) \lor R(c,a))]$$
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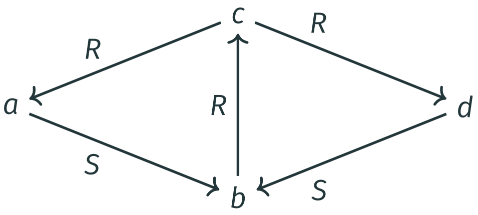

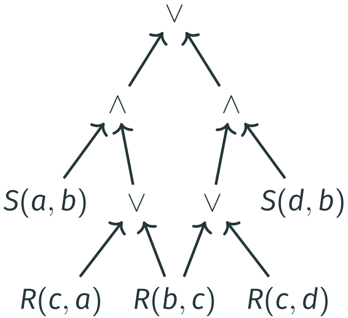

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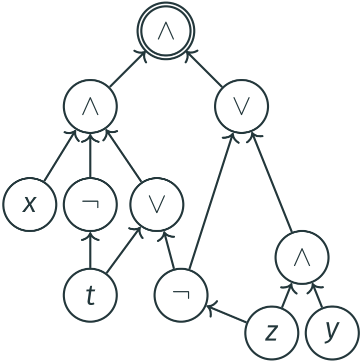
Question: what are the links between the two?

Treewidth and **d-SDNNFs**

Bounded treewidth Boolean circuits



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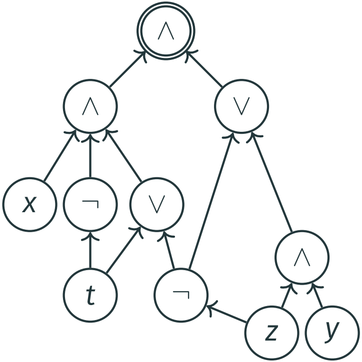

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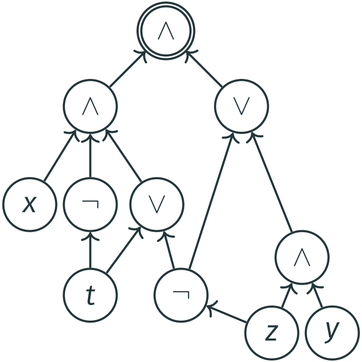

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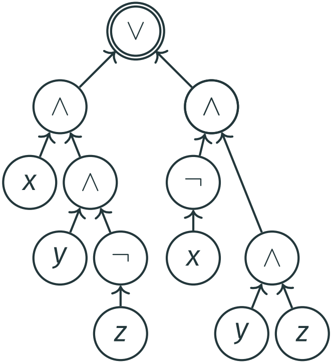
We can do message passing:

Theorem (Lauritzen & Spielgelhalter, 1988)

Fix $k \in \mathbb{N}$. Given a Boolean circuit C of treewidth $\leqslant k$, we can compute its probability in time $O(f(k) \times |C|)$, where f is singly exponential

d-SDNNF



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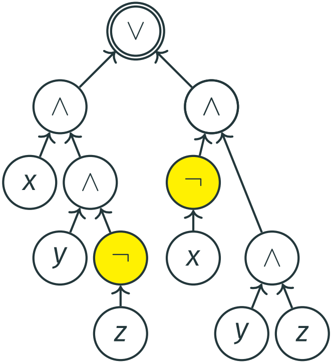

- Negation Normal Form: negations only applied to the leaves

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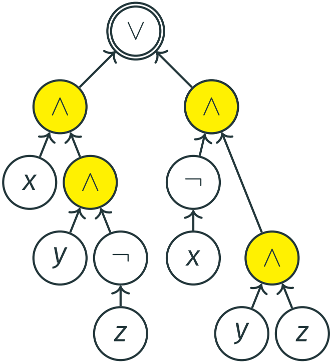

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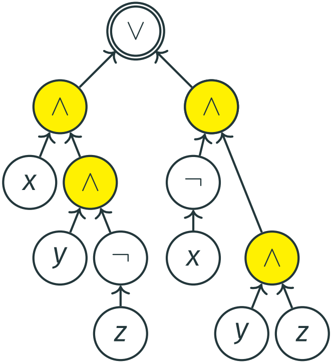

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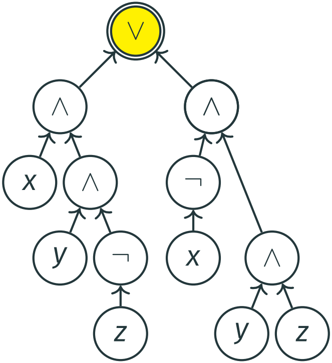

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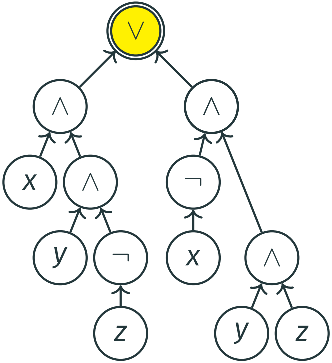

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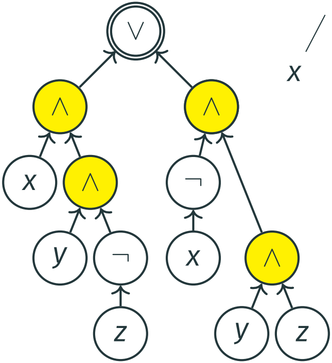

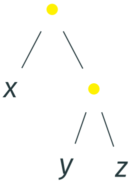

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Treewidth and d-SDNNFs: Upper bound

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*Let C be a Boolean circuit of **treewidth** $\leqslant k$.*
***We can compute** a **d-SDNNF** equivalent to C in time $O(|C| \times f(k))$,*
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→ Recaptures message passing through the use of knowledge compilation

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- The bound is generic: it applies to *any* monotone DNF/CNF

Proof Sketch for CNFs (1/2)

Use the connection made in [Bova, Capelli & Mengel, 2016] between the notion of combinatorial rectangle in **communication complexity** and SDNNFs.

Definition

A (X, Y)-rectangle is a Boolean function $R : 2^{X \cup Y} \to \{0, 1\}$ that can be written as $R_X \wedge R_Y$, for some Boolean functions $R_X : 2^X \to \{0, 1\}$ and $R_Y : 2^Y \to \{0, 1\}$. A (X, Y)-rectangle cover of a function $f : 2^{X \cup Y} \to \{0, 1\}$ is a set $\{R_1, \cdots, R_n\}$ of (X, Y)-rectangles such that $f \equiv \bigvee_{i=1}^{n} R_i$.

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Theorem (Bova, Capelli & Mengel, 2016)

Let C be an SDNNF computing a function φ on variables V, structured by a v-tree T. Let $n \in T$, and let (X, Y) be the partition of V that n induces. Then φ has a (X, Y)-rectangle cover of size at most $|C|$.

Proof Sketch for CNFs (2/2)

A CNF having no small rectangle cover:

Theorem (Sherstov, 2014)
Let $X = \{x_1, \ldots, x_n\}$ *and* $Y = \{y_1, \ldots, y_n\}$ *be two disjoint sets of variables. Then any* (X, Y)*-rectangle cover of the Boolean function* $\mathrm{SCOV}_n(X, Y) := \bigwedge_{i=1}^{n} x_i \vee y_i$ *has size* $\geqslant 2^n$.

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→ Rephrase treewidth as treesplitwidth, a new measure capturing the 'performance' of a v-tree

Application to PQE

CFG-Datalog program P of body-size ≤ k_P

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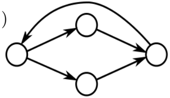
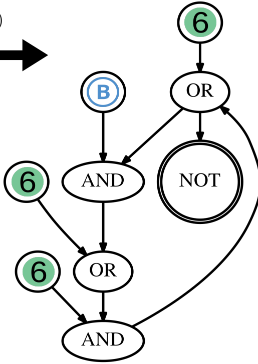
$O(g'(k_P,k_D)|P|)$

Two-way Alternating Tree Automaton A



$O(|A||E|)$

Provenance Cycluit



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(Paris Metro map)

$O(g(k_D)|D|)$

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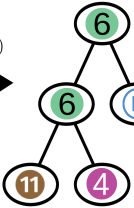

"Under which conditions is it impossible to go from station Corvisart to station Châtelet with the subway?"

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Cycluit
size $O(|P| \times |D|)$
treewidth $O(|P|)$

Application to PQE

Cycluit		Circuit
size $O(\|P\| \times \|D\|)$	▸	size $O(2^{\|P\|^\alpha} \times \|D\|)$
treewidth $O(\|P\|)$		treewidth $O(2^{\|P\|^\alpha})$

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▸

Circuit
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treewidth $O(2^{|P|^\alpha})$

▸

d-SDNNF
size $O(2^{2^{|P|^\alpha}} \times |D|)$

Application to PQE

Cycluit		Circuit		d-SDNNF
size $O(\|P\| \times \|D\|)$ treewidth $O(\|P\|)$	▸	size $O(2^{\|P\|^\alpha} \times \|D\|)$ treewidth $O(2^{\|P\|^\alpha})$	▸	size $O(2^{2^{\|P\|^\alpha}} \times \|D\|)$

Theorem

*Fix k_P and k_I. We can solve PQE of a **CFG-Datalog program P** on a **treelike database D** in time $O(2^{2^{|P|^\alpha}} |D|)$.*

Application to PQE

Cycluit
size $O(|P| \times |D|)$
treewidth $O(|P|)$

▸

Circuit
size $O(2^{|P|^\alpha} \times |D|)$
treewidth $O(2^{|P|^\alpha})$

▸

d-SDNNF
size $O(2^{2^{|P|^\alpha}} \times |D|)$

Theorem
*Fix k_P and k_I. We can solve PQE of a **CFG-Datalog program P** on a **treelike database D** in time $O(2^{2^{|P|^\alpha}} |D|)$.*

- 2EXP, but still better than previous nonelementary bounds

Conclusion (1/2)

Main contributions:

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1. Detailed study of the combined complexity of PQE of conjunctive queries on binary signatures

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Conclusion (1/2)

Main contributions:

1. Detailed study of the combined complexity of PQE of conjunctive queries on binary signatures
2. Efficient provenance computation for a new expressive query language (CFG-Datalog) on treelike data, introduction of a new provenance representation (cycluits)
3. Connections between two classes of Boolean circuits in knowledge compilation: width-based and semantics-based. Application to PQE of CFG-Datalog

Conclusion (2/2)

Ideas for Future work:

Conclusion (2/2)

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- Correlations?

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- Practical implementations?

Thanks for your attention!

CFG-Datalog: Definition by example

- Fragment of Datalog with stratified negation

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$$\begin{cases} \vdots \\ S_3(x,y,t) \leftarrow R_1(x,y) \land S_3(y,t,y) \land S_2(x,t) \land \neg S_1(x,z) \\ \vdots \\ \text{Goal}() \leftarrow \cdots \end{cases}$$

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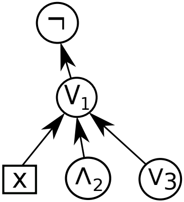
body-size $= \mathrm{MaxArity}(\sigma) \times \max_{\text{rule } r} \mathrm{NbAtoms}(r)$

"*size to write a rule*"

Construction sketch for slide 36



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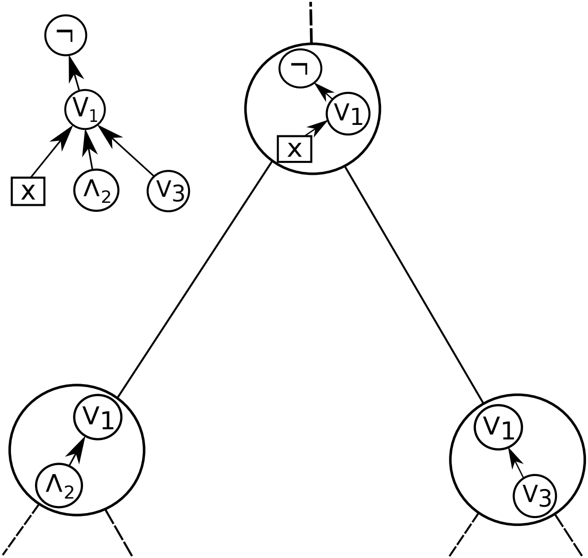

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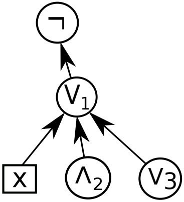

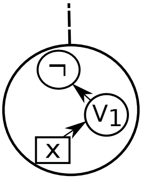

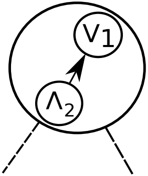

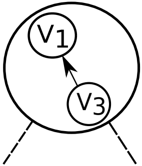

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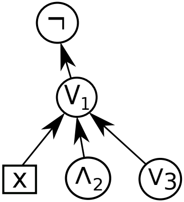

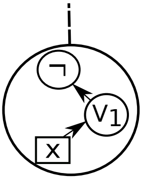

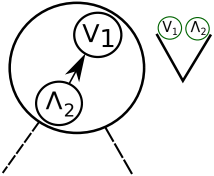

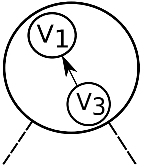

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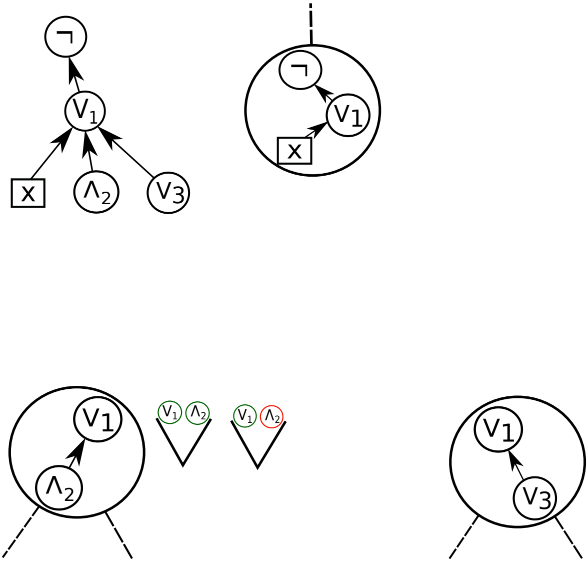

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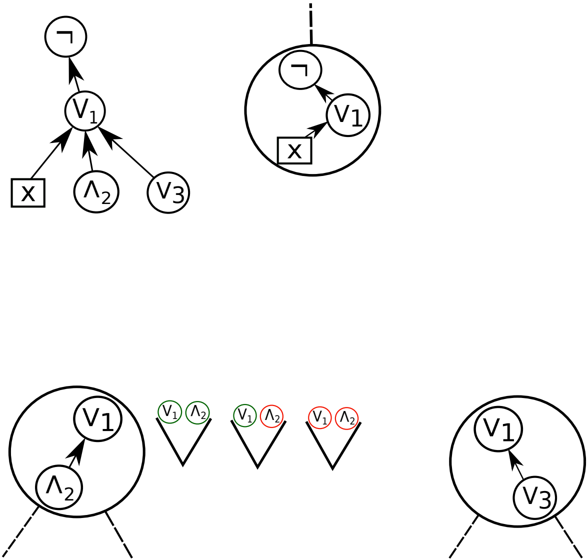

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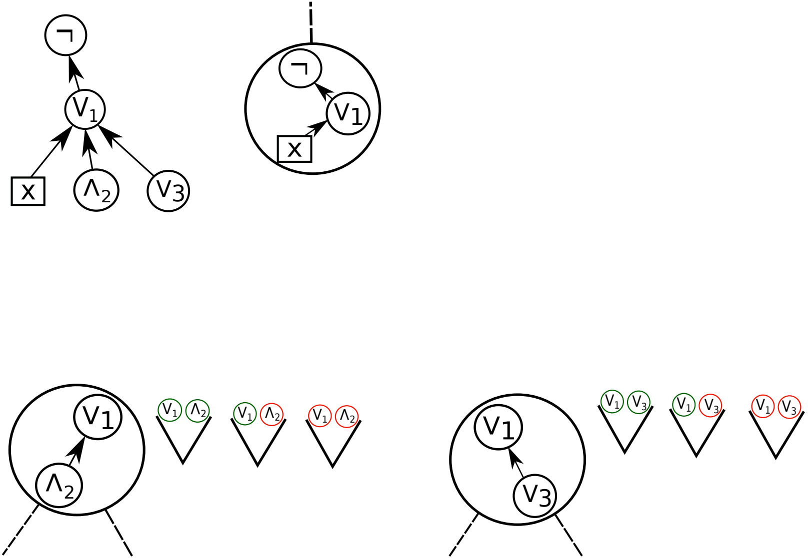

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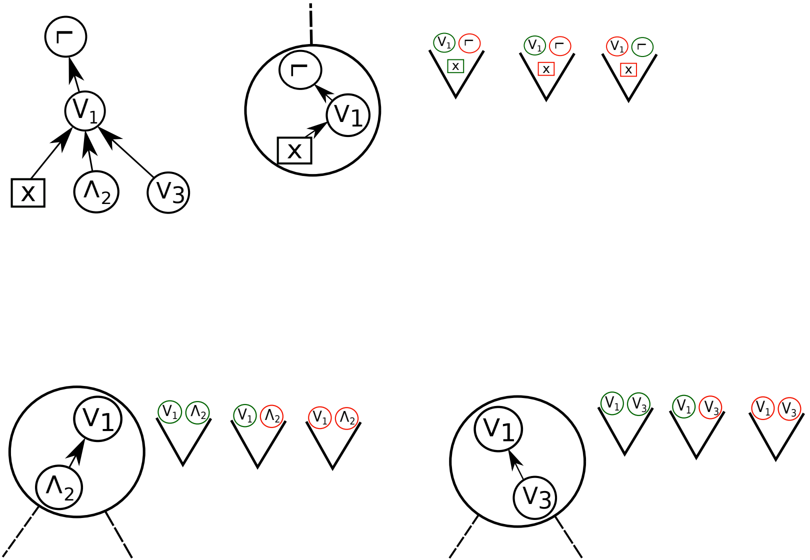

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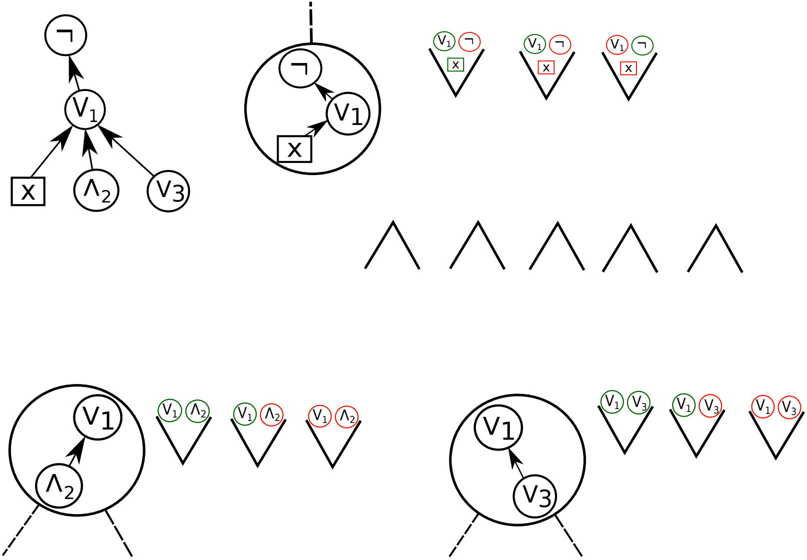

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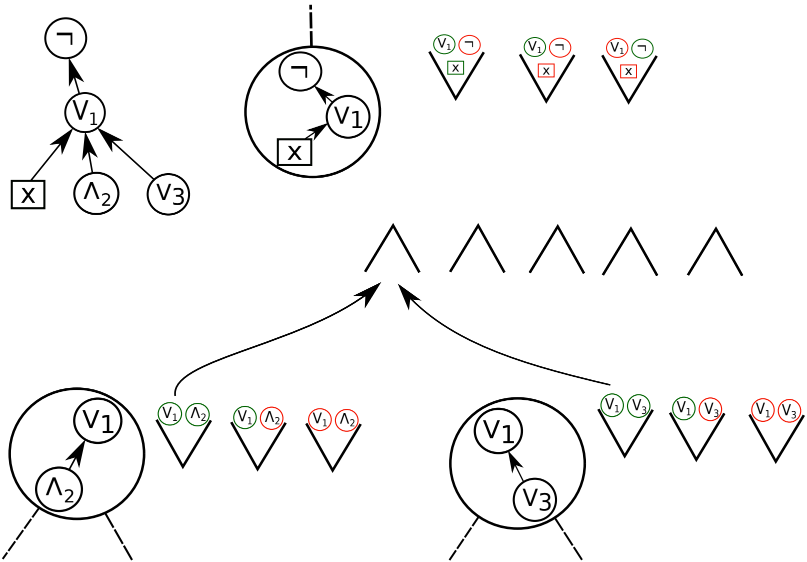

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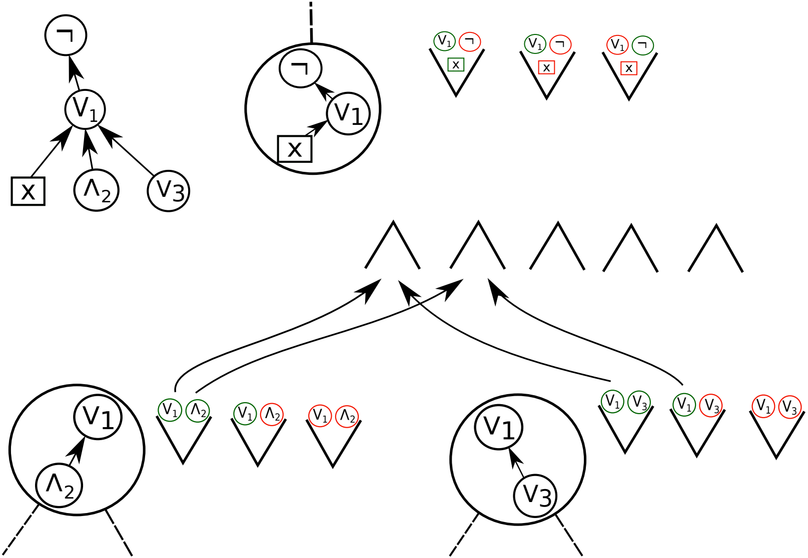

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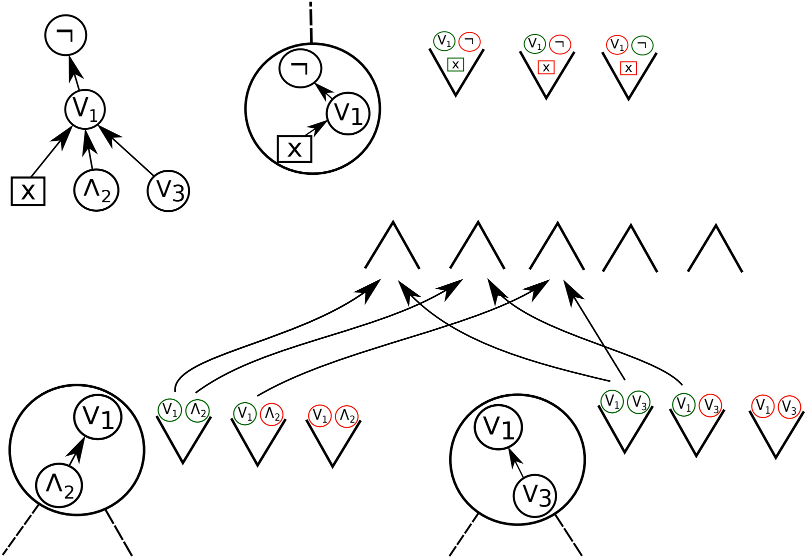

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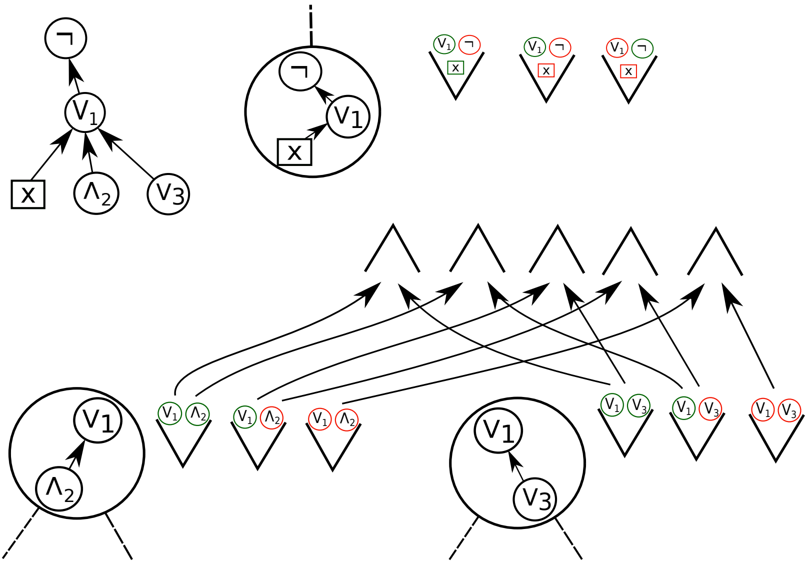

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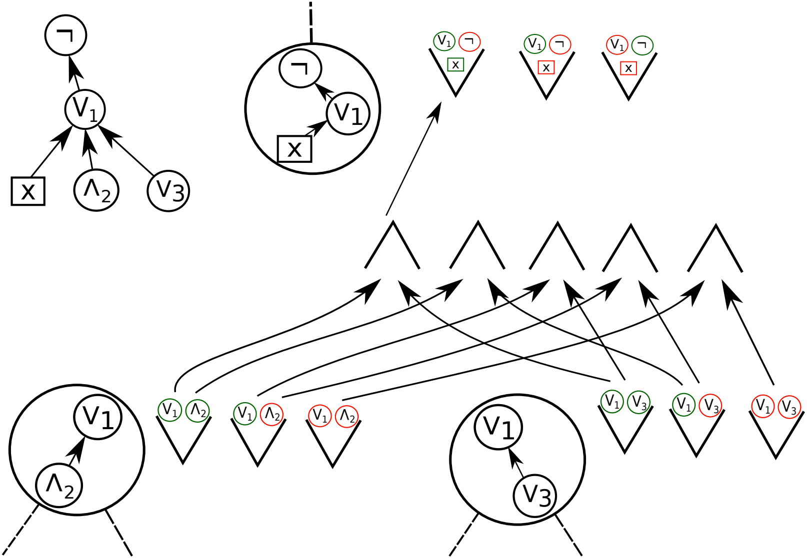

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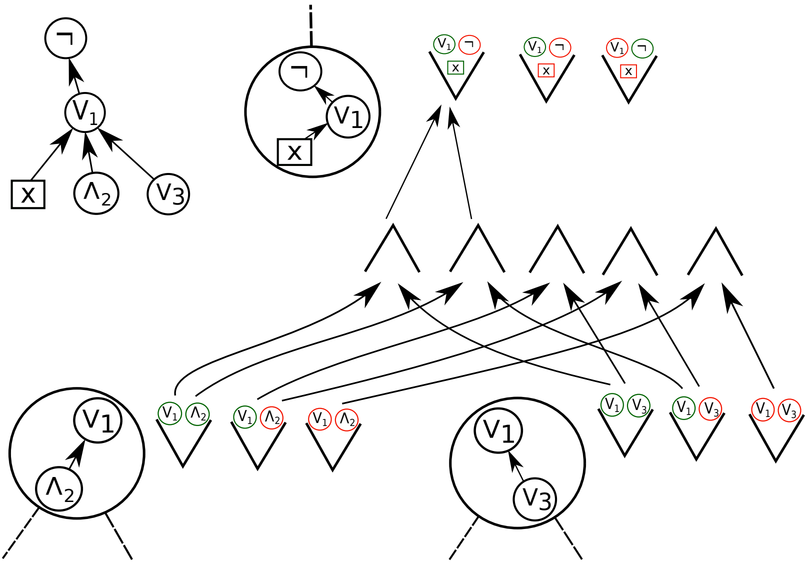

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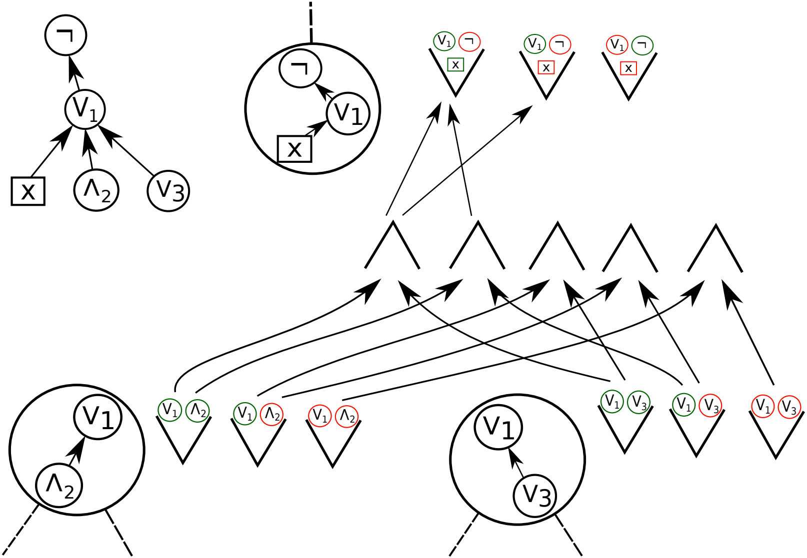

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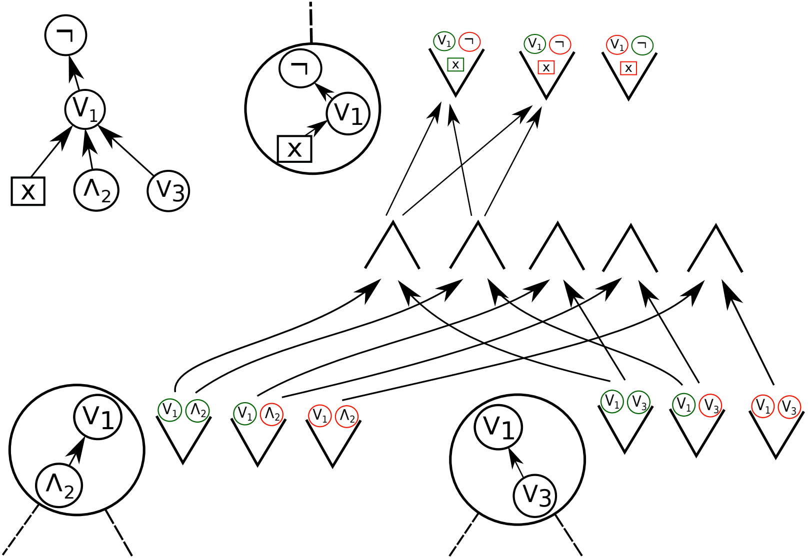

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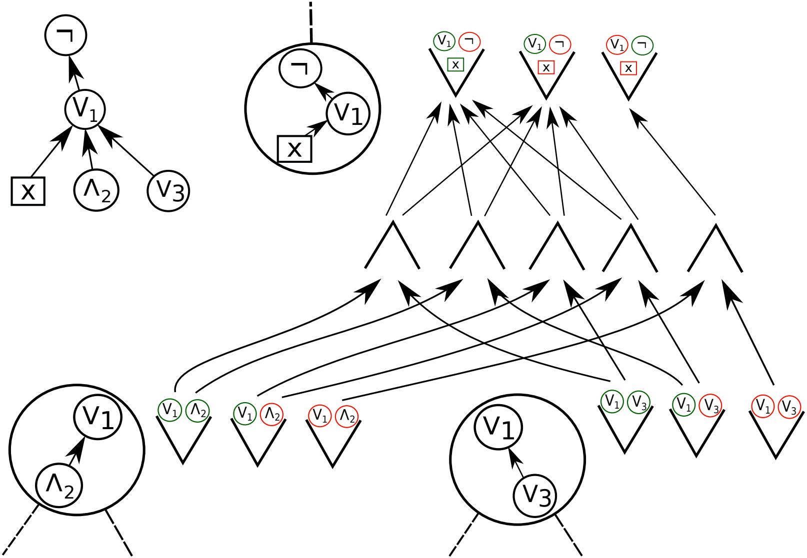

Reduction for $\mathcal{Q}$ = *one-way paths*, $\mathcal{I}$ = *polytrees*

$\varphi = X_1 Y_2 \vee X_1 Y_1 \vee X_2 Y_2$

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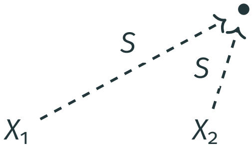

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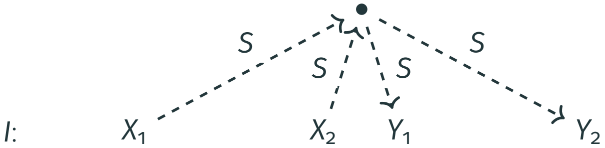

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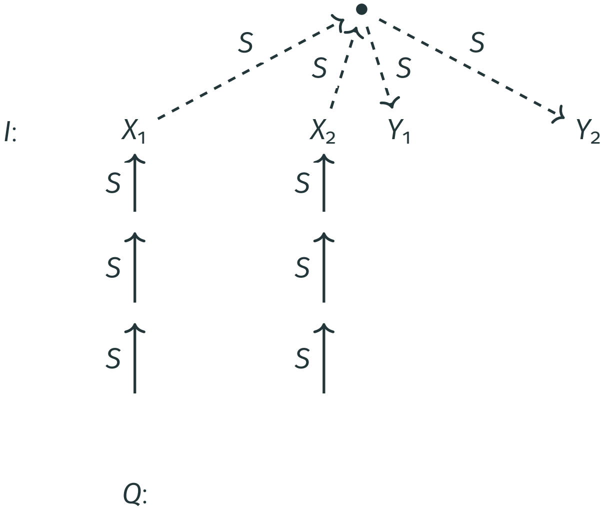

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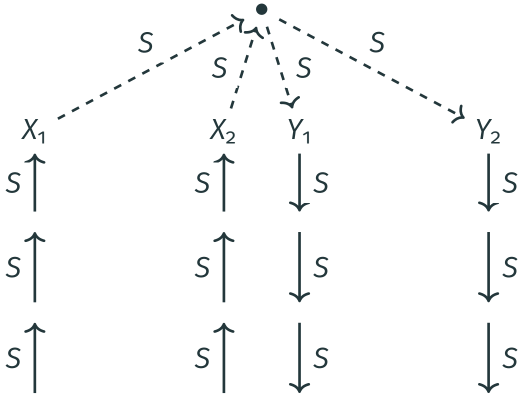

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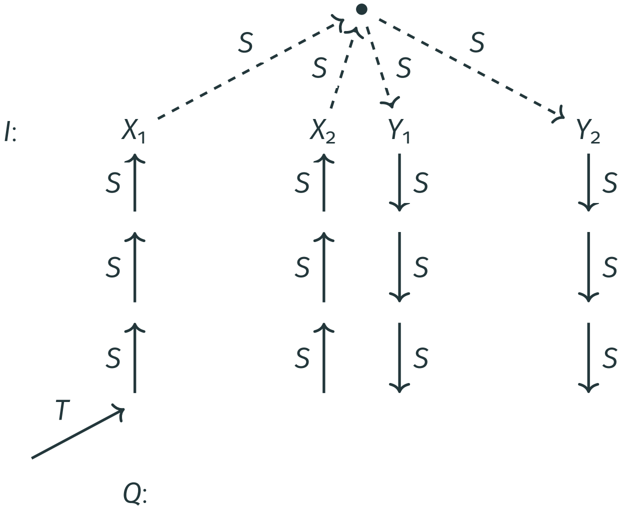

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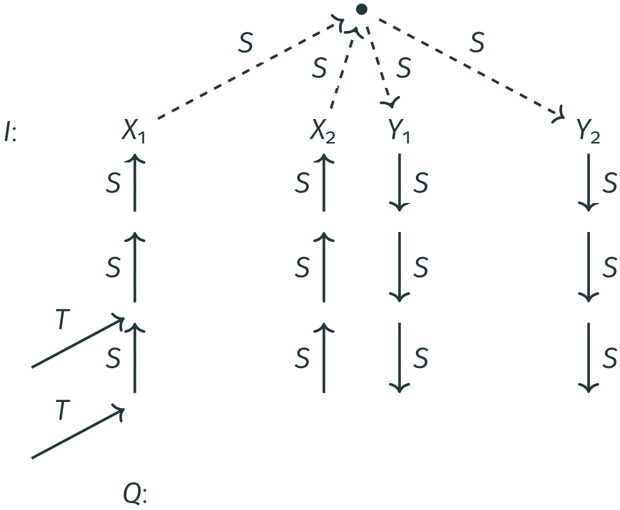

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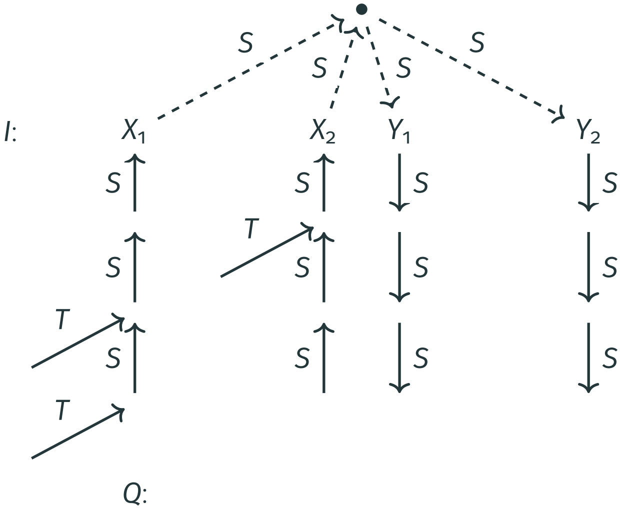

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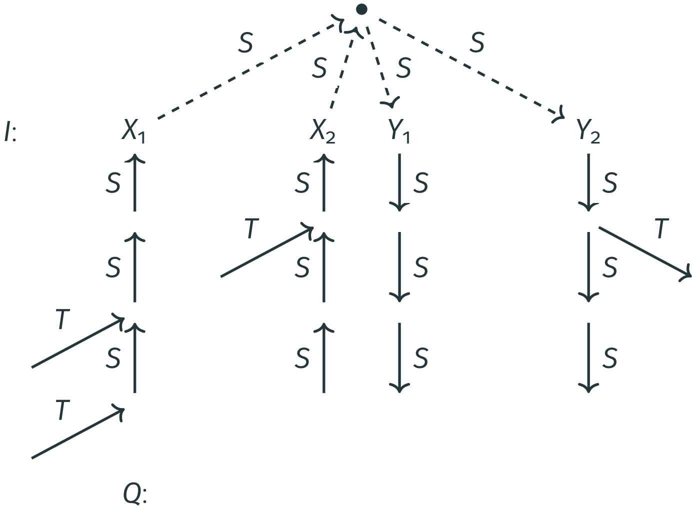

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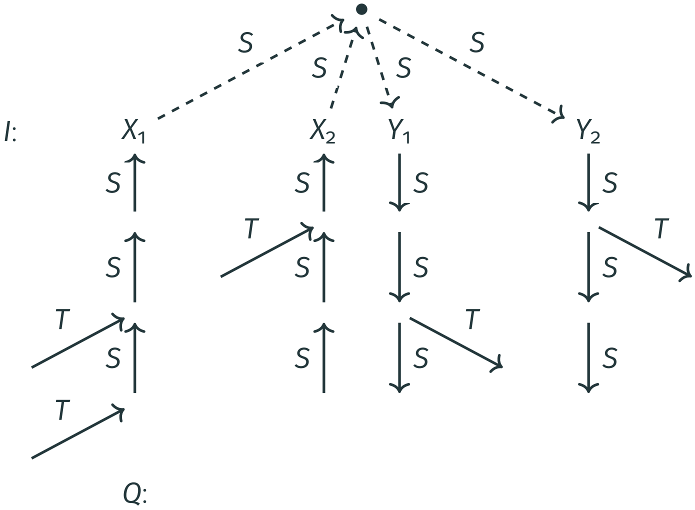

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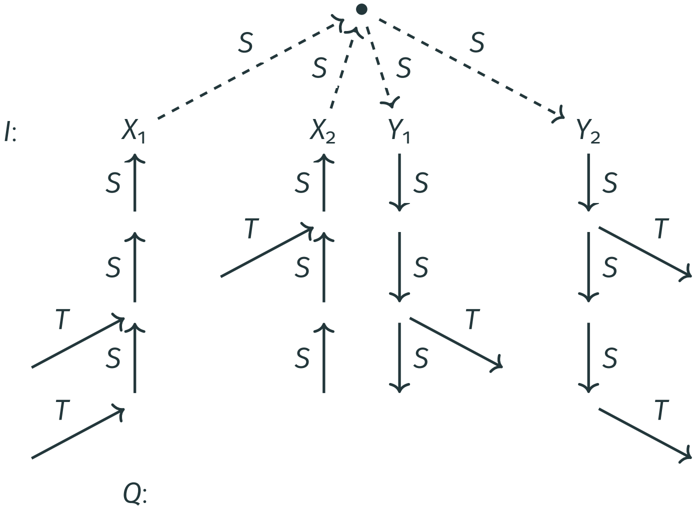

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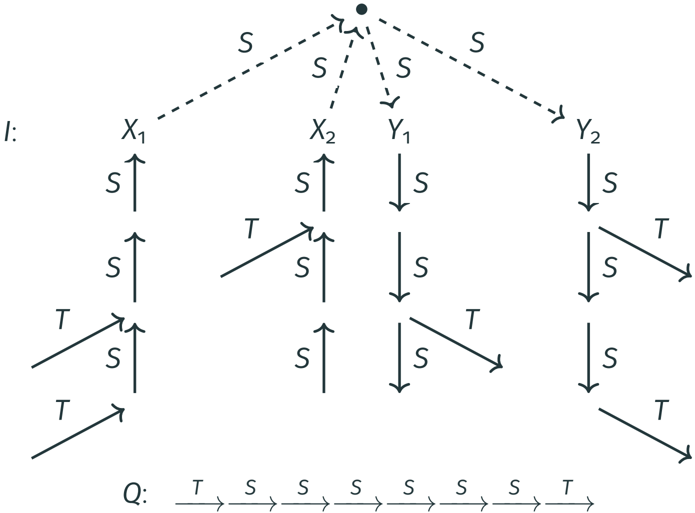

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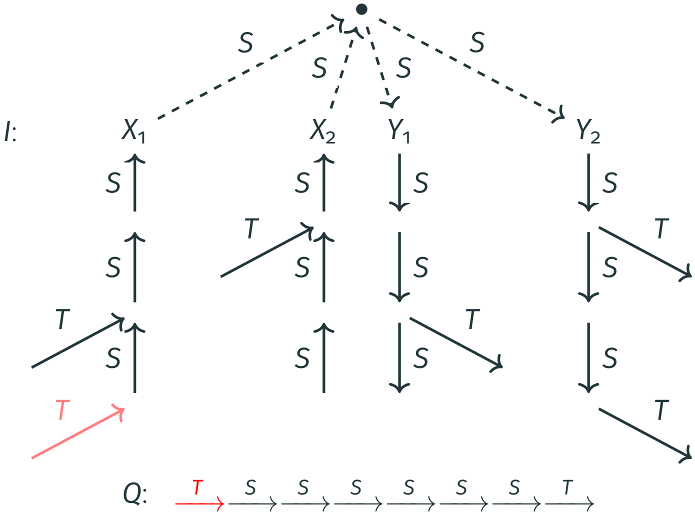

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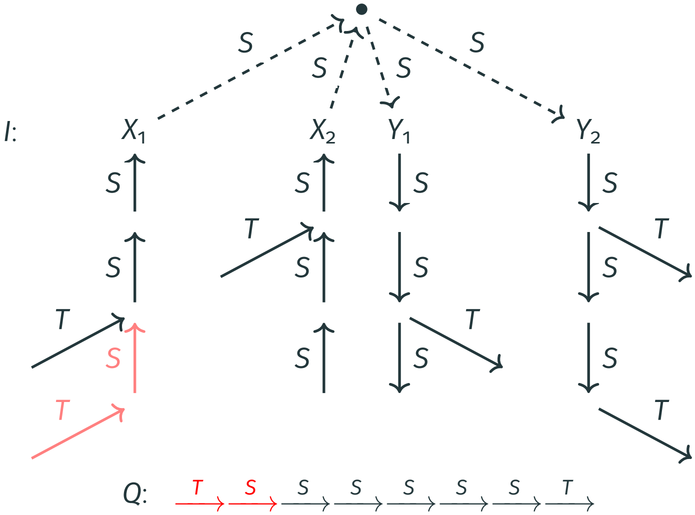

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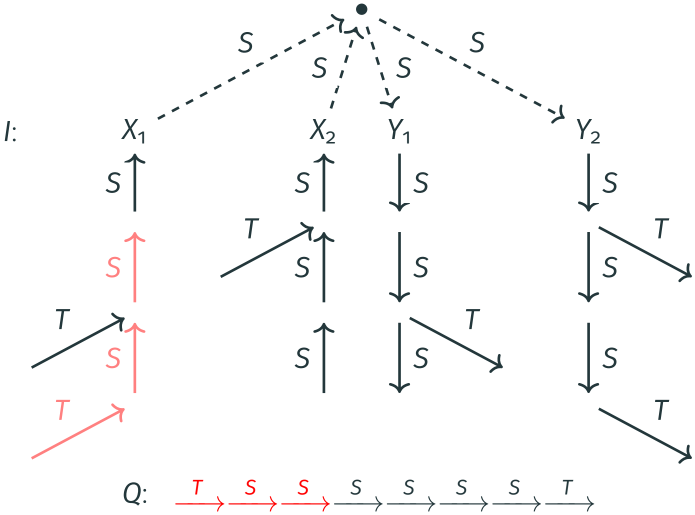

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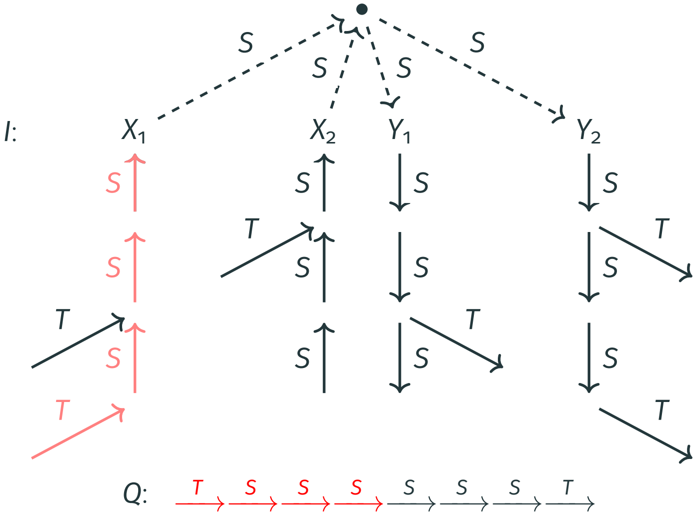

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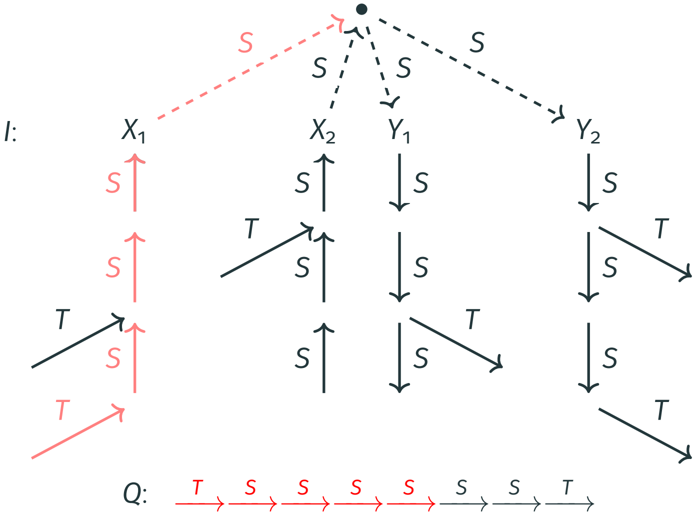

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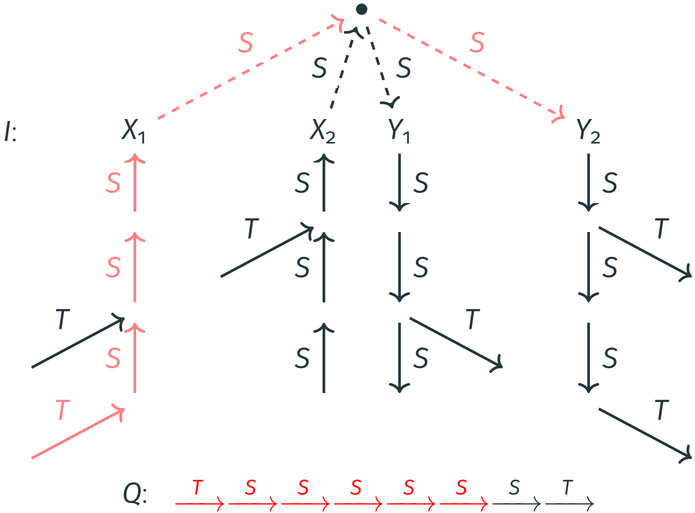

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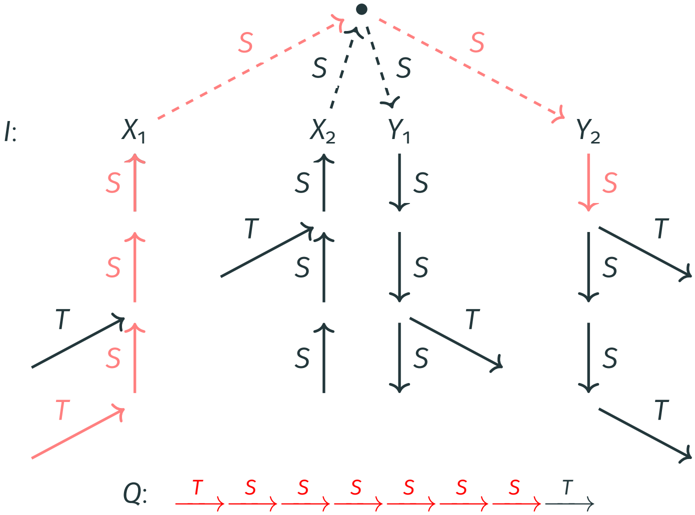

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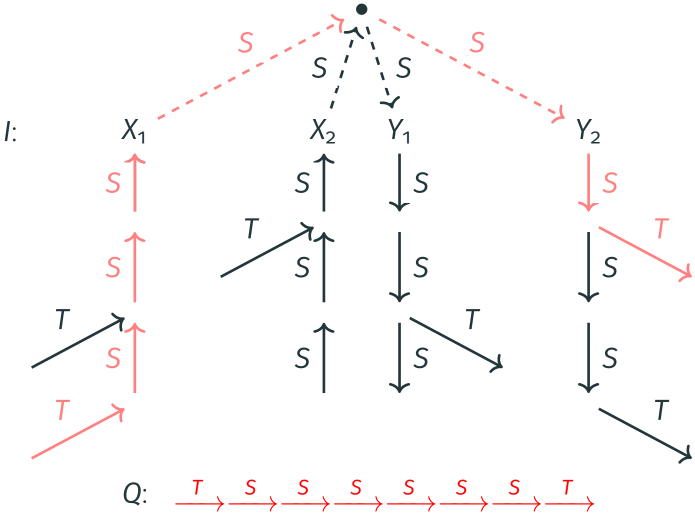

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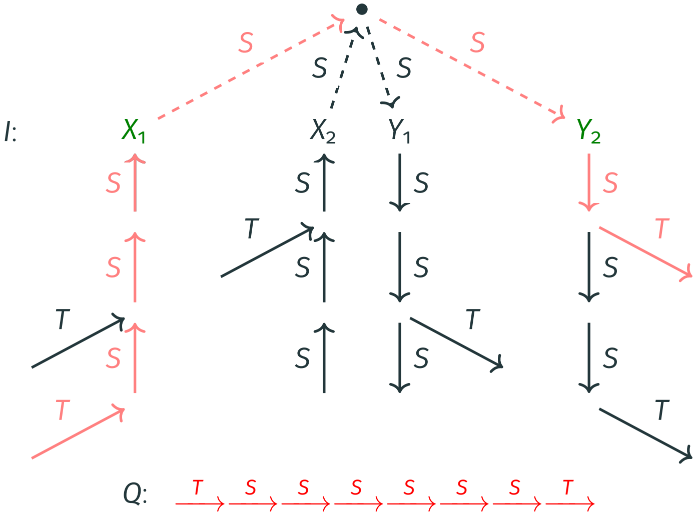

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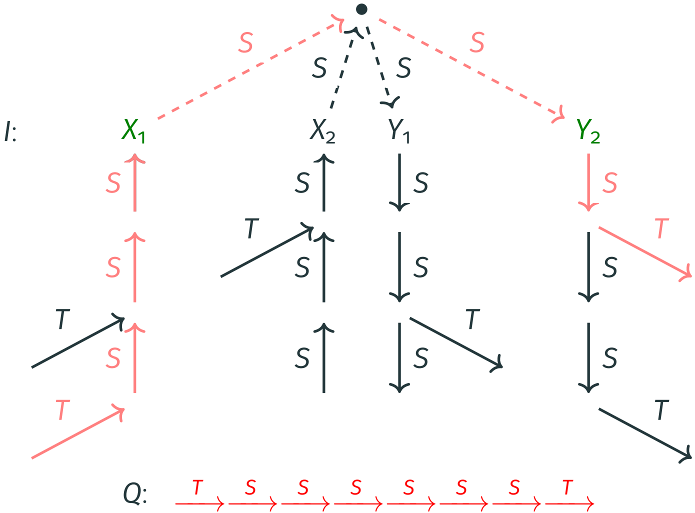

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$\#\varphi = \Pr((I, \pi) \models Q) \times 2^{|\text{vars}(\varphi)|}$