Counting Problems over Incomplete Databases

Mikaël Monet

Formal Methods team seminar at LaBRI

Setpember 29th, 2020

About me

[2012–2015] Engineering school in Nancy

[2015–2018] PhD in Paris (*Télécom ParisTech*) with **Pierre Senellart** and **Antoine Amarilli**

→ Database theory, uncertain data management

[2019–August 2020] Postdoctorate in Santiago de Chile (*IMFD*) with **Pablo Barceló**

→ Database theory, uncertain data management, logical aspects of machine learning, complexity of explainability tasks (AI)

[September] Off

[1st October] Research position at Inria Lille, team LINKS

Uncertain data management

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- But what if we **actually need/want to acknowledge this uncertainty**? (e.g, if querying the data without taking the uncertainty into account could lead to incorrect answers)

→ Need to develop theories, tools, etc. to be able to **represent** and **query** such uncertain data

- → This is uncertain data management!

Frameworks for uncertain data management

Lots of existing frameworks to represent and query uncertain data:

- Bayesian networks
- Markov random fields
- Graphical models
- Possibility theory, fuzzy logic, etc.

In this talk, focus on frameworks for relational databases:

- **Probabilistic databases**
- **Incomplete databases**

Probabilistic databases: example

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  - → simplest formalism: tuple-independent database

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Likes		π
Alice	Bob	0.5
Alice	John	1
Bob	Bob	0.2
John	Bob	0.7

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$\Pr(D') = (1 - 0.5) \times 1 \times (1 - 0.2) \times 0.7$

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$\Pr((D, \pi) \models q) = \sum_{\substack{D' \subseteq D \\ D \models q}} \Pr(D')$

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$$+ (1-0.5) \times [0.2 \times 0.7]$$

Incomplete databases: example

- Probabilistic databases: nice, but this is not what is used in practice most of the time...

ProductId	ProductName	Price	Color	Localisation
439	Printer	\$100	NULL	Paris center
782	Mouse	\$10	red	NULL
398	Mouse	\$30	red	Miami center
...	...	...	...	...

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→ Incomplete databases: relational databases with missing values

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Certain answers: (a) and (b)

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No certain answer :(

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  - → induces a notion of **best answer**
  - → also, **we can compare (some) tuples**

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  - → we can answer queries quantitatively (similar to probabilistic databases)

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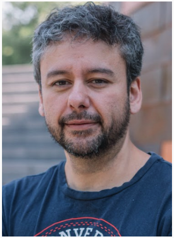
→ **This is what we'll do in this talk!**

My co-authors

Rest of the talk is based on paper "Counting Problems over Incomplete Databases" [PODS'20] with Marcelo Arenas and Pablo Barceló



Setting

- Incomplete databases with named (marked) nulls
- Each null $\perp$ comes with its own finite domain $\mathrm{dom}(\perp)$; all valuations ν are such that $\nu(\perp) \in \mathrm{dom}(\perp)$
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Problems studied

- Fix a Boolean query q

Definition: problem $\#\mathsf{Val}(q)$

Input: an incomplete database D, together with *finite* domains $\mathrm{dom}(\perp)$ for each null of D

Output: the number of valuations ν such that $\nu(D) \models q$

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Definition: problem $\#\mathsf{Comp}(q)$

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Output: the number of *completions* $\nu(D)$ such that $\nu(D) \models q$

Example

- **Example:** $D = \{S(a,b), S(\perp_1, a), S(a, \perp_2)\}$,
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$(\nu(\bot_1), \nu(\bot_2))$	(a,a)	(a,b)	(b,a)	(b,b)	(c,a)	(c,b)
$\nu(D)$	$\begin{array}{cc}\hline \multicolumn{2}{c}{S} \\ \hline a & b \\ a & a \\ \hline\end{array}$	$\begin{array}{cc}\hline \multicolumn{2}{c}{S} \\ \hline a & b \\ a & a \\ \hline\end{array}$	$\begin{array}{cc}\hline \multicolumn{2}{c}{S} \\ \hline a & b \\ b & a \\ a & a \\ \hline\end{array}$	$\begin{array}{cc}\hline \multicolumn{2}{c}{S} \\ \hline a & b \\ b & a \\ \hline\end{array}$	$\begin{array}{cc}\hline \multicolumn{2}{c}{S} \\ \hline a & b \\ c & a \\ a & a \\ \hline\end{array}$	$\begin{array}{cc}\hline \multicolumn{2}{c}{S} \\ \hline a & b \\ c & a \\ \hline\end{array}$
$\nu(D) \vDash Q$?	Yes	Yes	Yes	No	Yes	No

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$(\nu(\perp_1), \nu(\perp_2))$	(a,a)	(a,b)	(b,a)	(b,b)	(c,a)	(c,b)
$\nu(D)$	S: (a, b), (a, a)	S: (a, b), (a, a)	S: (a, b), (b, a), (a, a)	S: (a, b), (b, a)	S: (a, b), (c, a), (a, a)	S: (a, b), (c, a)
$\nu(D) \models Q$?	Yes	Yes	Yes	No	Yes	No

4 satisfying valuations, 3 satisfying completions

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$\nu(D) \models Q$?	Yes	Yes	Yes	No	Yes	No

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$\rightarrow$ Study the complexity of these problems depending on q (*data complexity*). Obtain **dichotomies**? Can we efficiently **approximate** the number of solutions? Etc.

Problems variants and query language

We also study the settings where:

- all labeled nulls are distinct (***Codd tables***; by contrast to ***naïve tables***)
- all nulls share the same domain (***uniform setting***)

→ In total we consider 8 **different settings** $(\{\#\mathrm{Val}, \#\mathrm{Comp}\} \times \{\text{naïve/Codd}\} \times \{\text{non-uniform/uniform}\})$

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- We focus only on **self-join free Boolean conjunctive queries** ($\mathrm{sjfBCQs}$)

Outline

The dichotomies for exact counting

Counting valuations vs. counting completions

Approximations

The dichotomies for exact counting

Patterns in sjfBCQs

Definition: pattern

A sjfBCQ q' is a pattern of another sjfBCQ q if q' can be obtained from q by deleting atoms or variable occurrences, and then reordering the variables inside the atoms and renaming (injectively) the variables and relation names

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Example: (from now on all variables are existentially quantified)
$q' = R'(u, u, y) \land S(z)$ is a pattern
of $q = R(u, x, u) \land S(y, y) \land T(x, s, z, s)$

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$\rightarrow R(u, x, u) \wedge S(y)$ (delete a variable occurrence)

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- $\rightarrow$ $R(u,x,u) \wedge S(y)$ (delete a variable occurrence)
- $\rightarrow$ $R(u,u,x) \wedge S(y)$ (reorder variables occurrences)
- $\rightarrow$ $R'(u,u,x) \wedge S(y)$ (rename R into R')

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of $q = R(u, x, u) \wedge S(y, y) \wedge T(x, s, z, s)$

- $\rightarrow R(u, x, u) \wedge S(y, y)$ (delete third atom)
- $\rightarrow R(u, x, u) \wedge S(y)$ (delete a variable occurrence)
- $\rightarrow R(u, u, x) \wedge S(y)$ (reorder variables occurrences)
- $\rightarrow R'(u, u, x) \wedge S(y)$ (rename R into R')
- $\rightarrow R'(u, u, y) \wedge S(z)$ (rename x into y and y into z)

Patterns in sjfBCQs

Definition: pattern

A sjfBCQ q' is a pattern of another sjfBCQ q if q' can be obtained from q by deleting atoms or variable occurrences, and then reordering the variables inside the atoms and renaming (injectively) the variables and relation names

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Note: reordering and injective renaming are not important, it is just so that we can formally say things like:

- $R(x, y)$ is a pattern of $R(y, x)$; or
- $R(x)$ is a pattern of $S(y)$
- etc.

Proof strategy

Lemma

Let q, q' be $\mathrm{sjfBCQs}$ such that q' is a pattern of q. Then we have $\#\mathsf{Val}(q') \leq^{\mathrm{p}} \#\mathsf{Val}(q)$

Where $\leq^{\mathrm{p}}$ denote polynomial-time parsimonious reductions (and the same results holds for counting completions, and also if we restrict to Codd tables and/or to the uniform setting)

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→ for each of the 8 variants of the problem, find a set of patterns that are hard and such that if a sjfBCQ does not have any of these patterns then the problem is in PTIME

Example 1: $\#$Val, naïve, non-uniform

Consider counting valuations, naïve setting (named nulls that can appear in multiple places), non-uniform (each null $\perp$ comes with its own domain $\mathrm{dom}(\perp)$)

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  - → All variable occurrences are distinct, so every valuation is satisfying

Example 2: completions, naïve, Codd

Now consider counting completions for Codd databases (all nulls are distinct), non-uniform

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- In other words, here every sjfBCQ is hard...

The hard patterns

	Counting valuations		Counting completions	
	Non-uniform	**Uniform**	**Non-uniform**	**Uniform**
Naïve	$R(x,x)$ $R(x) \wedge S(x)$	$R(x,x)$ $R(x) \wedge S(x,y) \wedge T(y)$ $R(x,y) \wedge S(x,y)$	$R(x)$	$R(x,x)$ $R(x,y)$
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(So. . . not much is tractable)

Counting valuations vs. counting completions

When are our problems in $\#\mathrm{P}$?

- For a Boolean query q, let $\mathrm{MC}(q)$ denote the model checking problem for q

Fact

If $\mathrm{MC}(q)$ is PTIME then $\#\mathsf{Val}(q)$ is in $\#\mathrm{P}$.

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What about counting completions? In general when $\mathrm{MC}(q)$ is PTIME, is $\#\mathsf{Comp}(q)$ in $\#\mathrm{P}$? **Unlikely:**

Proposition

There exists an sjfBCQ q such that $\#\mathsf{Comp}(q)$ is not in $\#\mathrm{P}$ unless $\mathrm{NP} \subseteq \mathrm{SPP}$

A natural complexity class for counting completions (1/2)

- A counting problem A is in SpanP if there exists a nondeterministic transducer M (= Turing machine with output tape) running in polynomial time such that, on input x, the **number of distinct outputs** for $M(x)$ is equal to $A(x)$

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  - → (A problem in SpanP but unknown to be complete for it: INPUT: a graph G; OUTPUT: the number of Hamiltonian subgraphs of G)

A natural complexity class for counting completions (2/2)

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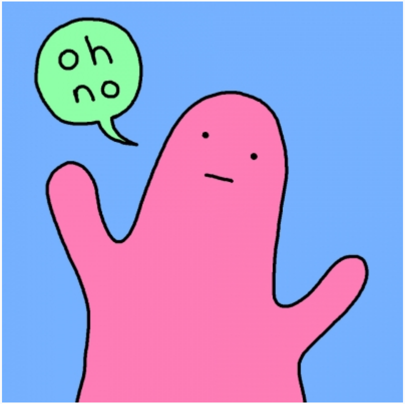
For Codd tables we can still show membership in $\#\mathrm{P}$:

Proposition

For **Codd tables**, if $\mathrm{MC}(q)$ is PTIME then $\#\mathsf{Comp}(q)$ is in $\#\mathrm{P}$

Approximations



My counting problem is very much intractable :(

→ Try Fully Polynomial-time Randomized Approximation Scheme!

Fully Polynomial-time Randomized Approximation Scheme!

Definition (FPRAS)

Let Σ be a finite alphabet and $f : \Sigma^* \to \mathbb{N}$ be a counting problem. Then f is said to have an FPRAS if there is a randomized algorithm $\mathcal{A} : \Sigma^* \times (0,1) \to \mathbb{N}$ and a polynomial $p(u,v)$ such that, given $x \in \Sigma^*$ and $\epsilon \in (0,1)$, algorithm $\mathcal{A}$ runs in time $p(|x|, 1/\epsilon)$ and satisfies the following condition:

$$\Pr\left(|f(x) - \mathcal{A}(x,\epsilon)| \leq \epsilon f(x)\right) \geq \frac{3}{4}.$$

Fully Polynomial-time Randomized Approximation Scheme!

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Note: the property of having an FPRAS is closed under polynomial-time parsimonious reductions (i.e., if we have an FPRAS for a counting problem A and for counting problem B we have that $B \leq^{\mathrm{p}} A$, then we also have an FPRAS for B).

FPRAS for counting valuations

Proposition

For every Boolean UCQ q, the problem $\#\mathsf{Val}(q)$ has a FPRAS

Proof: via SpanL. SpanL = there exists an NL transducer with write-only output tape such that the result is the number of distinct outputs

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Theorem (Arenas et al. [PODS'19])

Every problem in SpanL has an FPRAS

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FPRAS for counting completions?

Theorem (Dyer et al. [SICOMP'2002])

Counting vertex covers has no FPRAS unless $\mathrm{NP} = \mathrm{RP}$

- Our reduction from $\#\mathrm{VC}$ for Codd tables to $\#\mathsf{Comp}(\exists x\, R(x))$ was **parsimonious**
- Our reduction for the notion of pattern is also **parsimonious**

→ Therefore $\#\mathsf{Comp}(q)$ restricted to Codd tables for any sjfBCQ has no FPRAS unless $\mathrm{NP} = \mathrm{RP}$

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What about the uniform setting? We prove that for naïve tables, uniform setting, $\#\mathrm{Comp}(q)$ has no FPRAS if q contains a non-unary symbol (otherwise it is PTIME)

- For uniform Codd tables, **we do not know**

Conclusion

To sum up:

- Counting valuations and completions is **hard**, even in very restricted settings (uniform Codd tables)
- But counting valuations has a **FPRAS** for UCQs
- While counting completions does not
- SpanP is the right class to consider for problems of the form $\#\mathrm{Comp}(q)$
- If you liked it, we have a lot of cute reductions in the paper :)

Thanks for your attention!

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