# Expressive Power of Graph Neural Networks

Pablo Barceló[1], Egor Kostylev[2], **Mikaël Monet**[1], Jorge Pérez[1], Juan Reutter[1]

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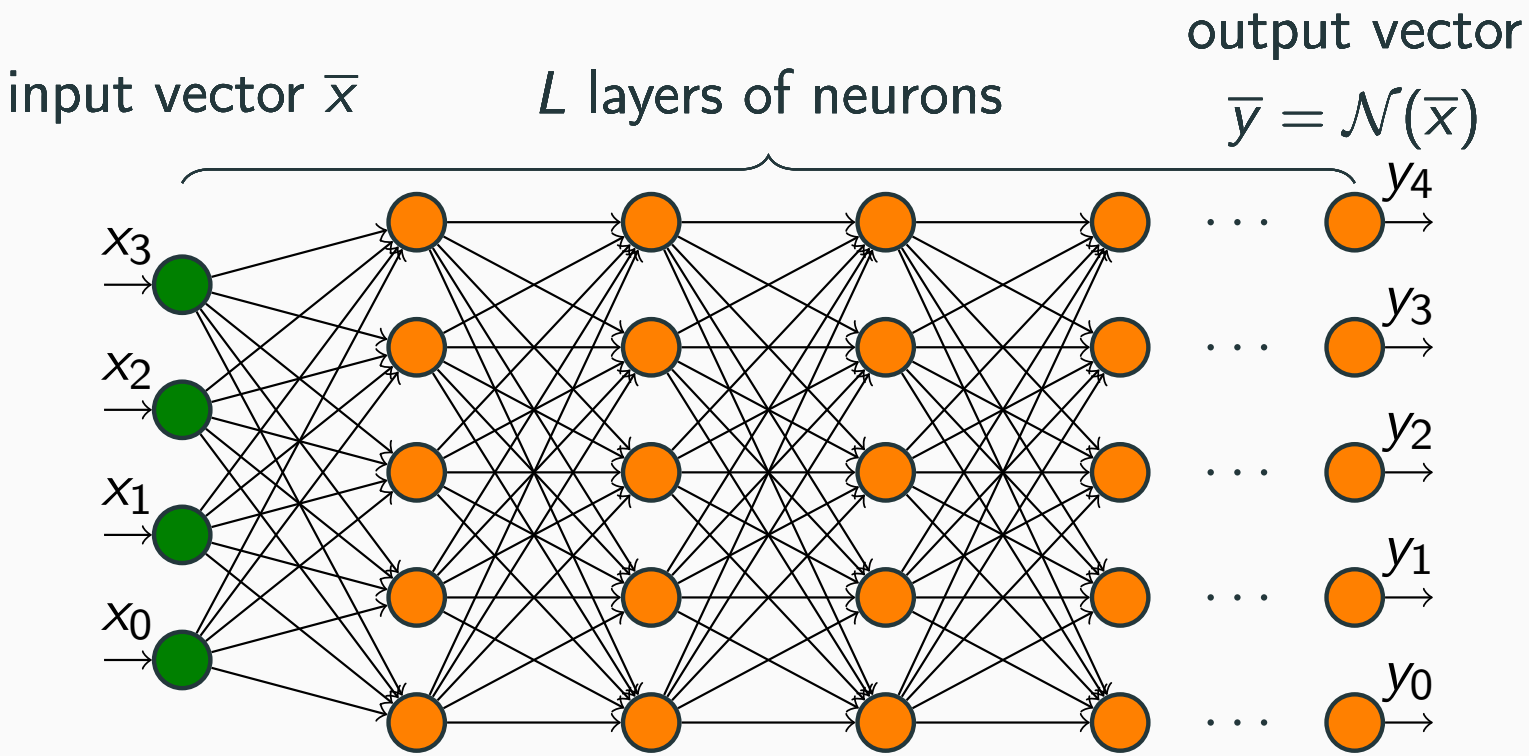
[1]Millennium Institute for Foundational Research on Data, Chile

[2]Oxford University, UK

# Motivation: why *graph* neural networks?

# Neural Networks (NNs)

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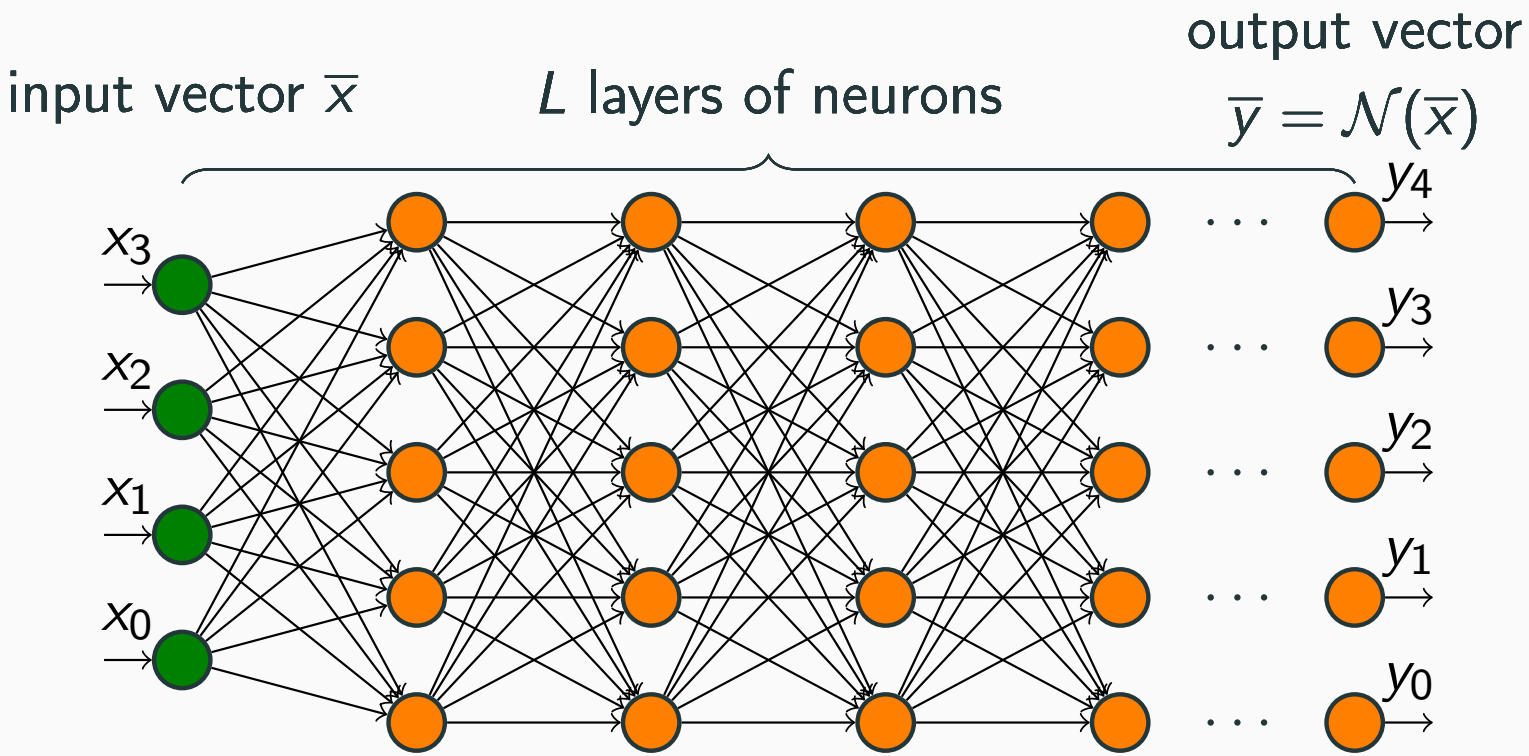


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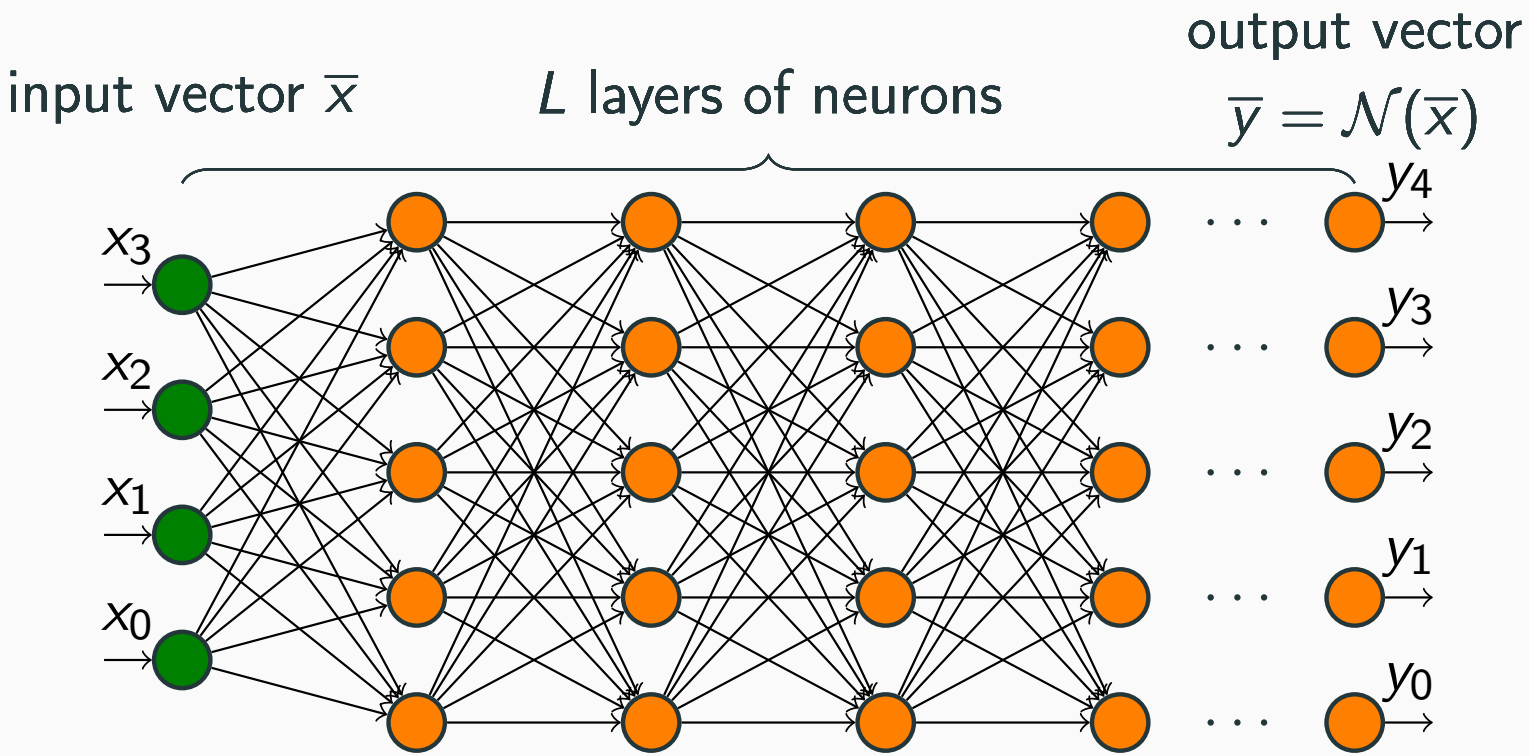


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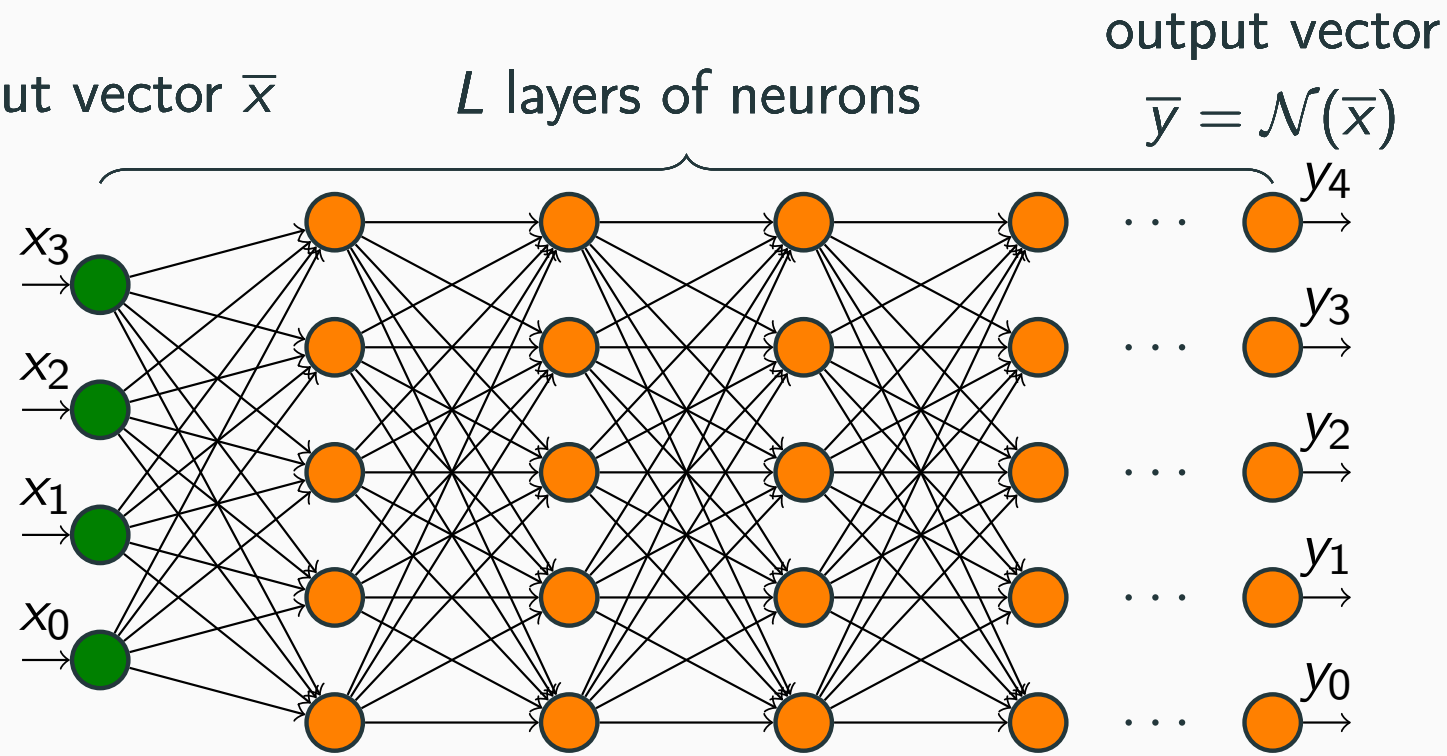


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- Goal: find the weights that "solve" your problem (classification, clustering, regression, etc.)

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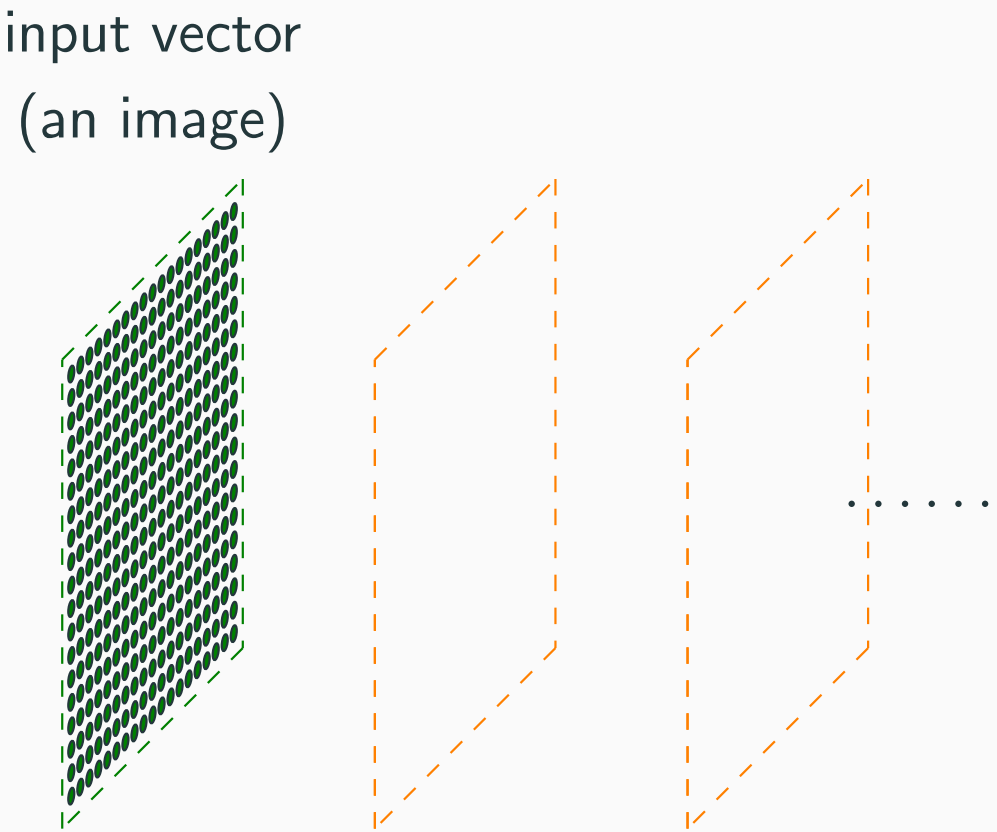
$\rightarrow$ example: input is a $250 \times 250$ pixels image, and we want to build a fully connected NN with 500 neurons per layer

$\rightarrow$ between each consecutive layers we have $250 \times 250 \times 500 = 31,250,000$ weights

# Convolutional Neural Networks

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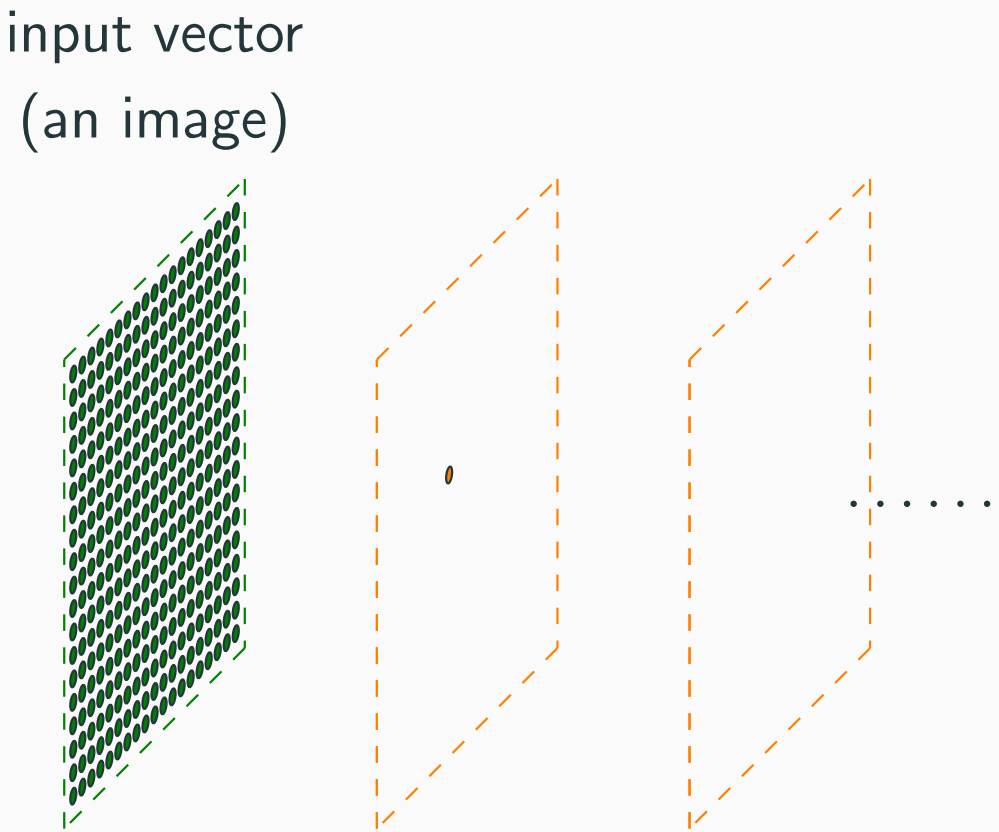


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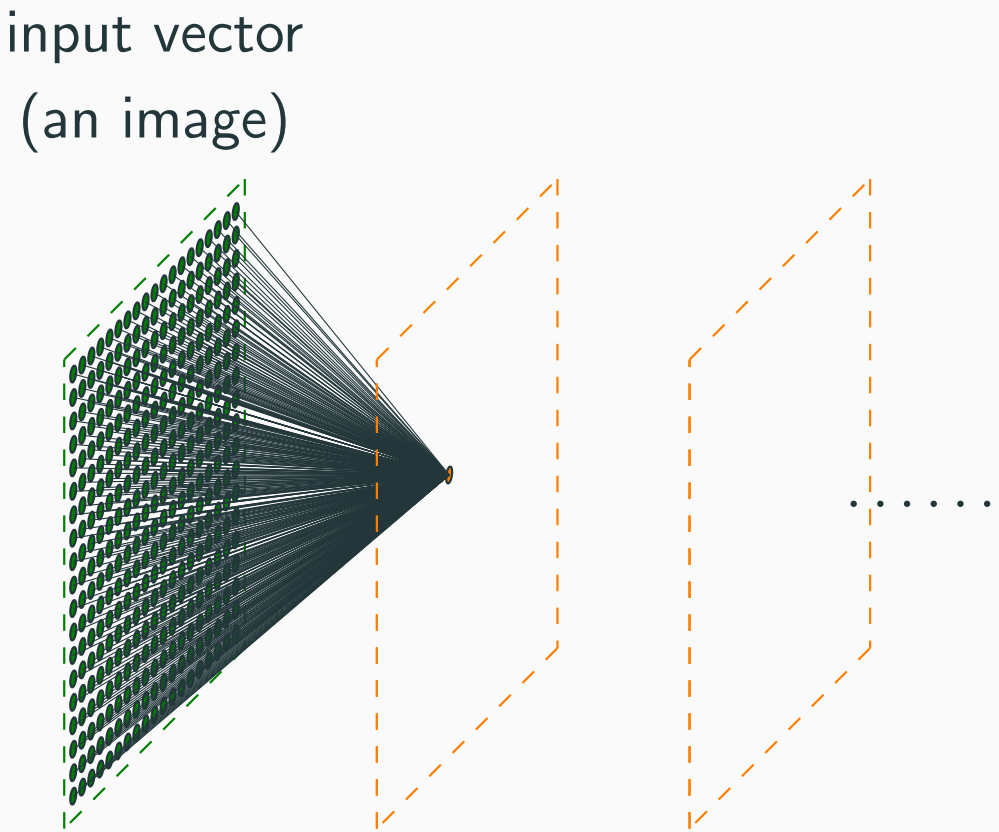


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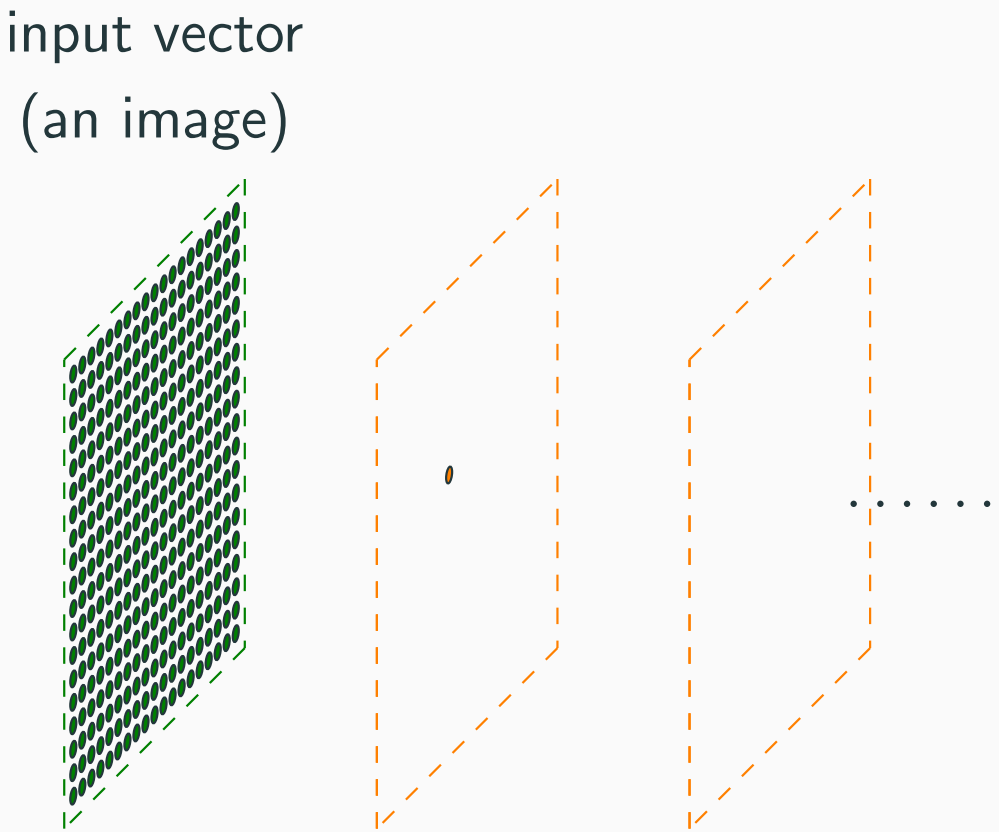


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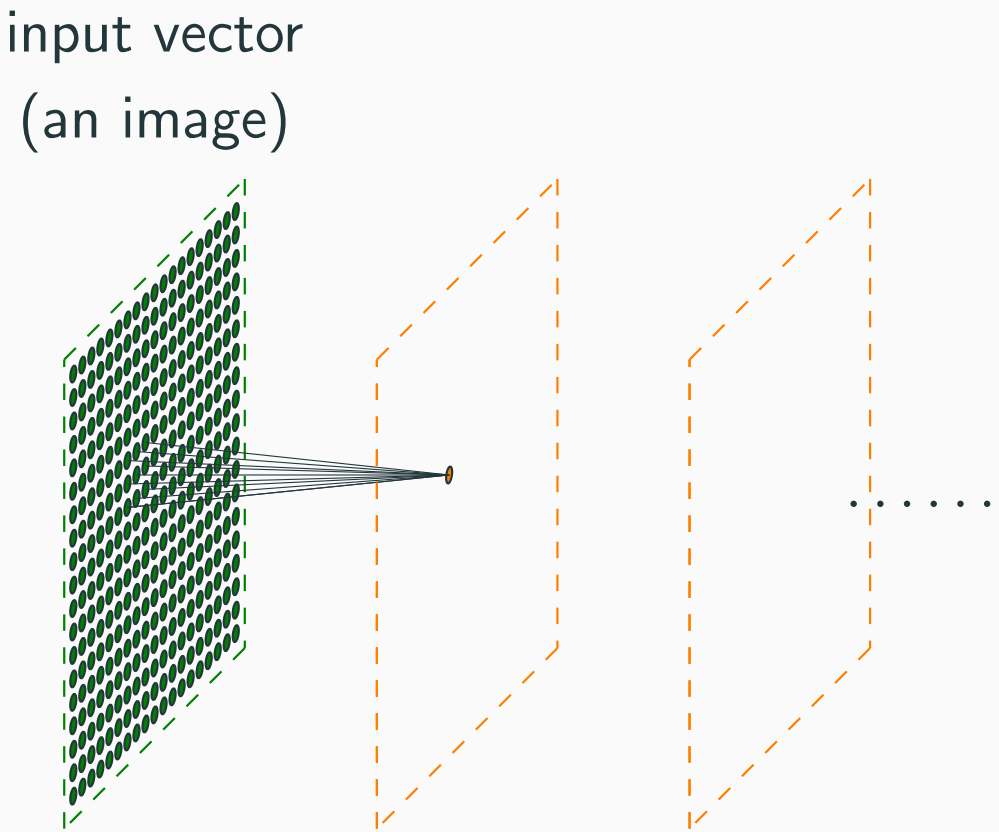


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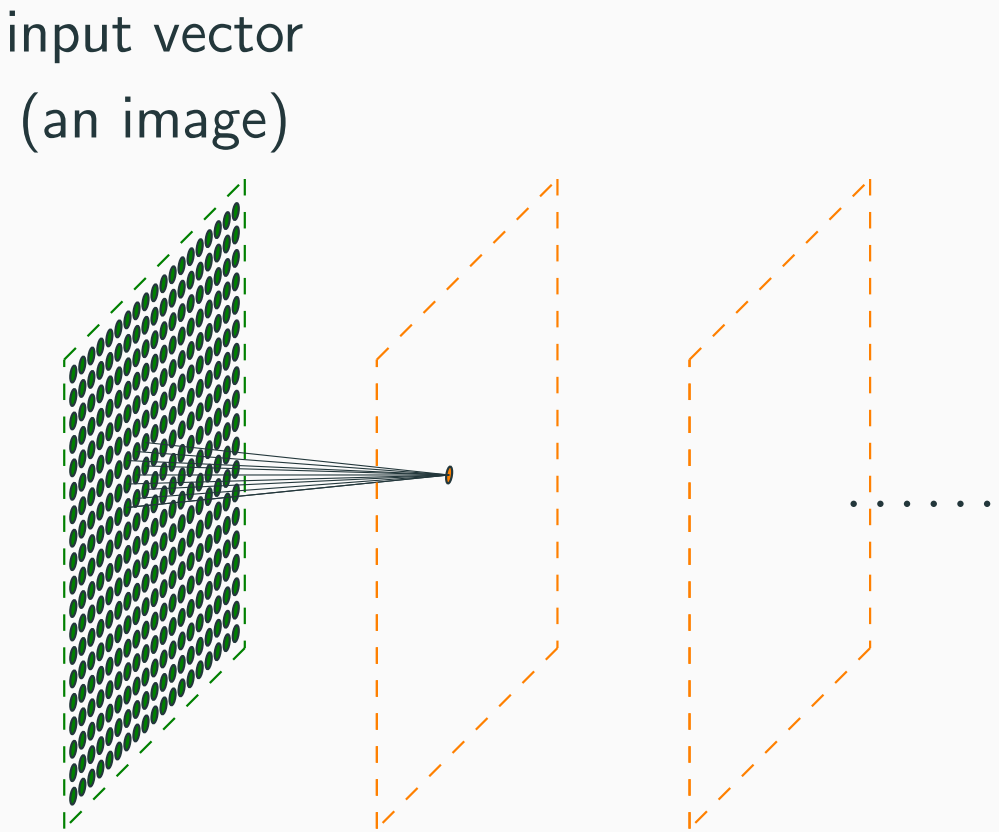


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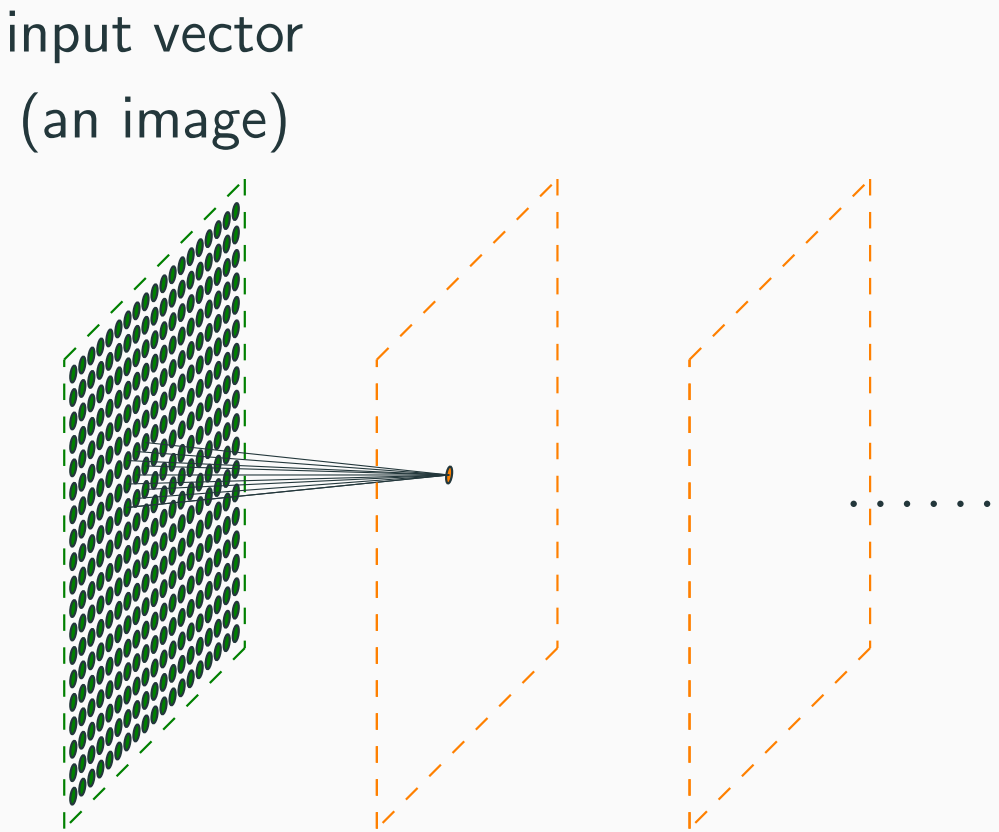


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$\rightarrow$ other advantage: recognize patterns that are local

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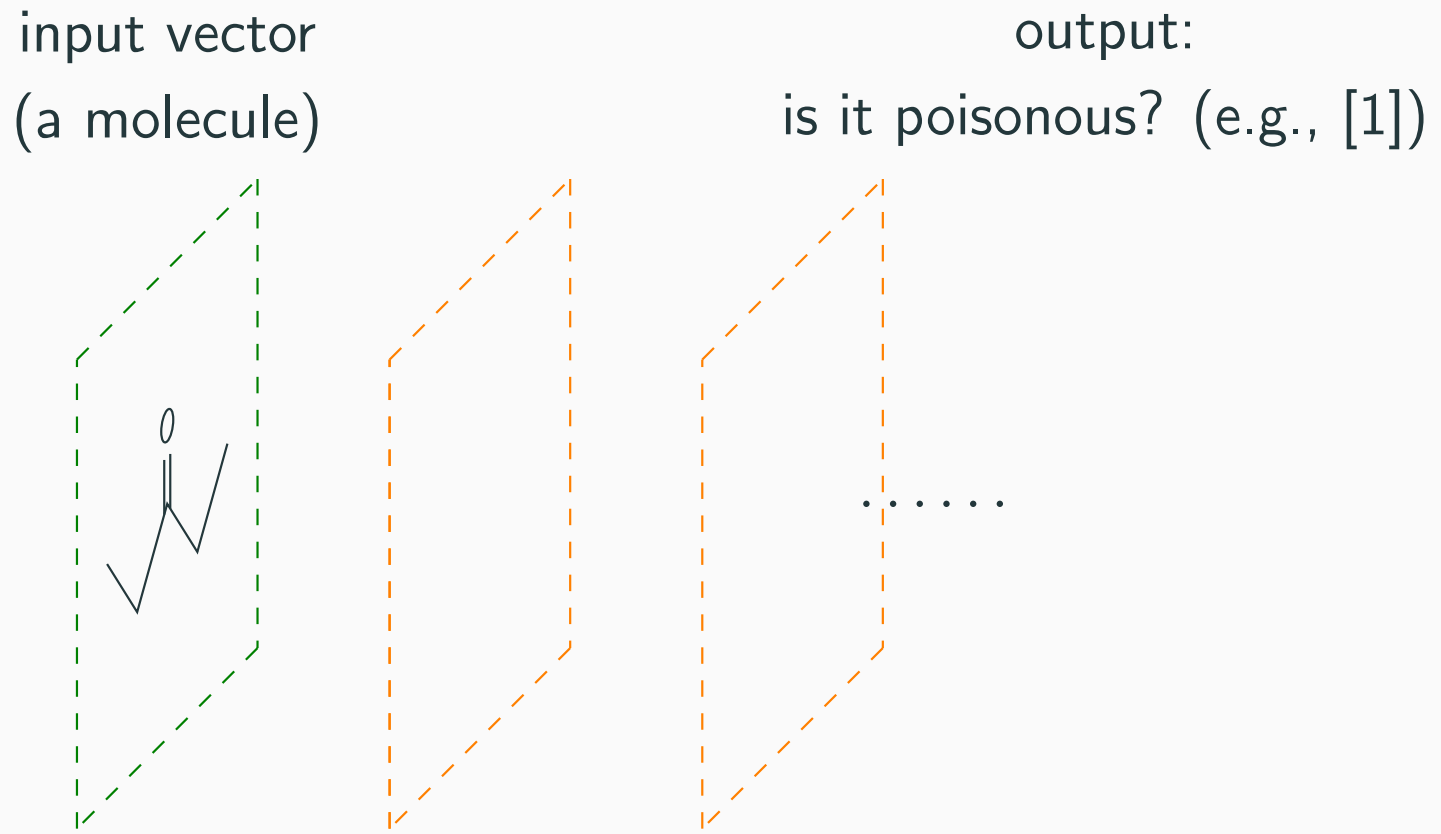


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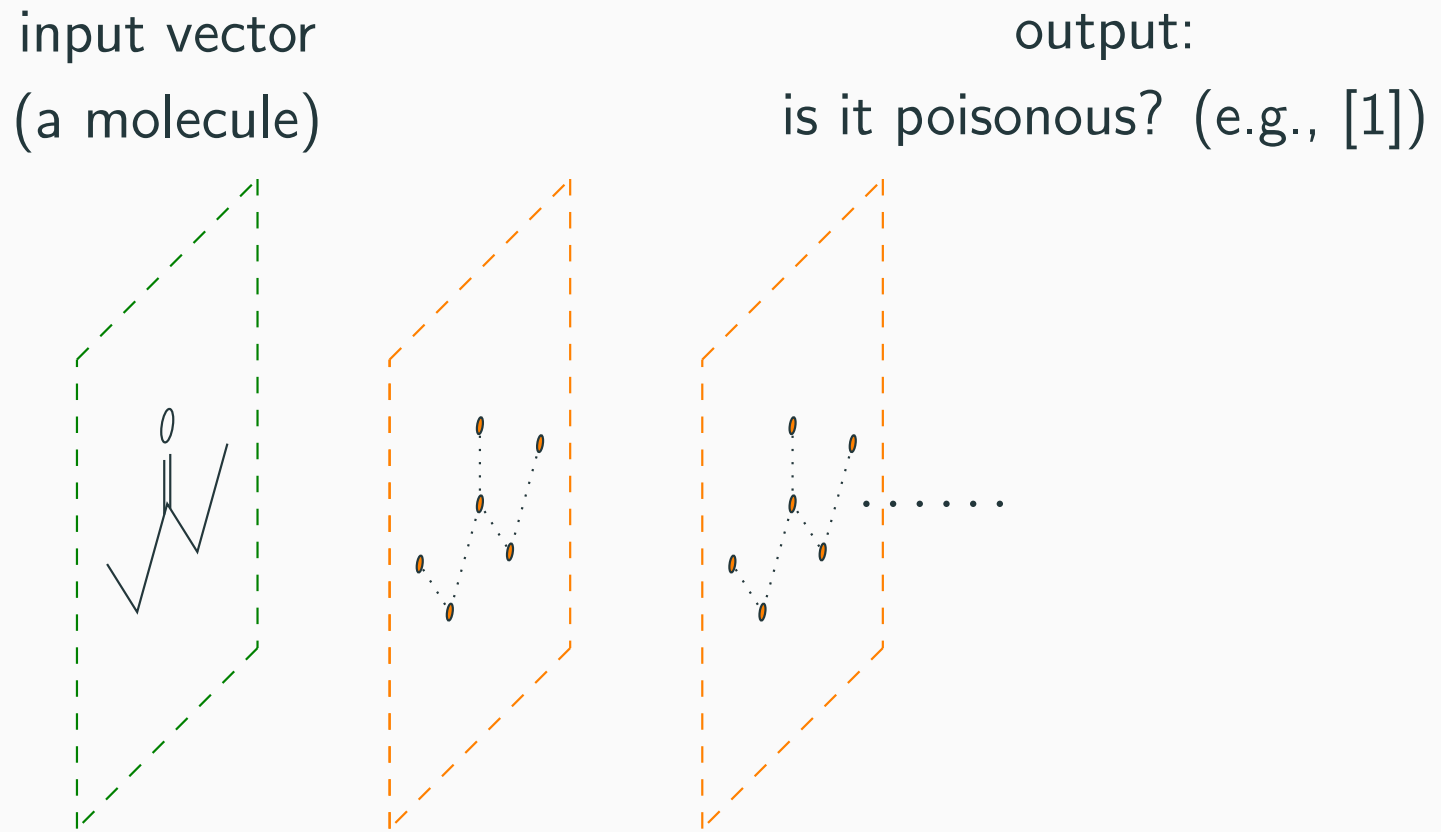


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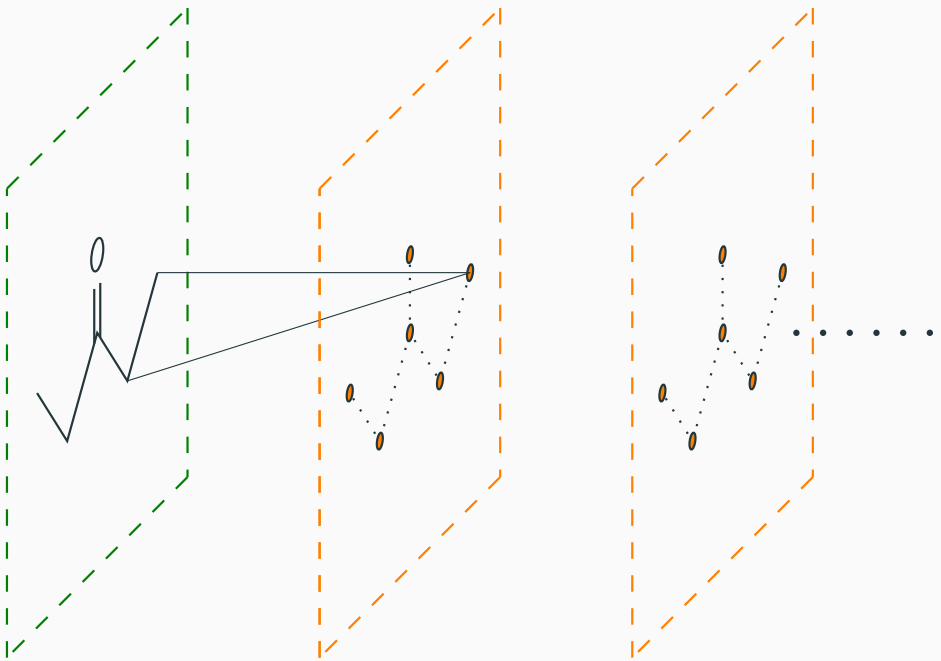

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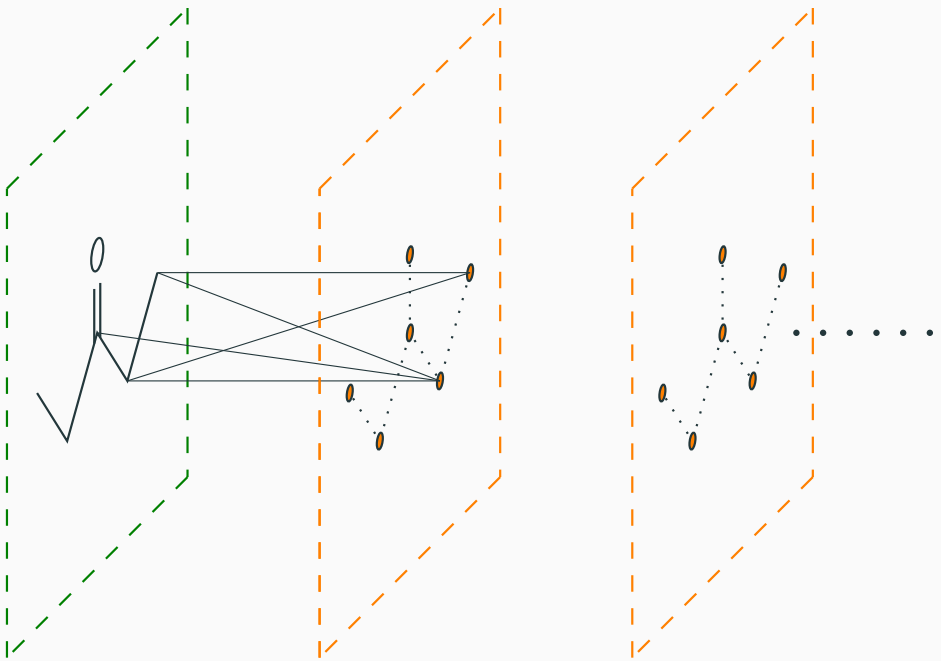

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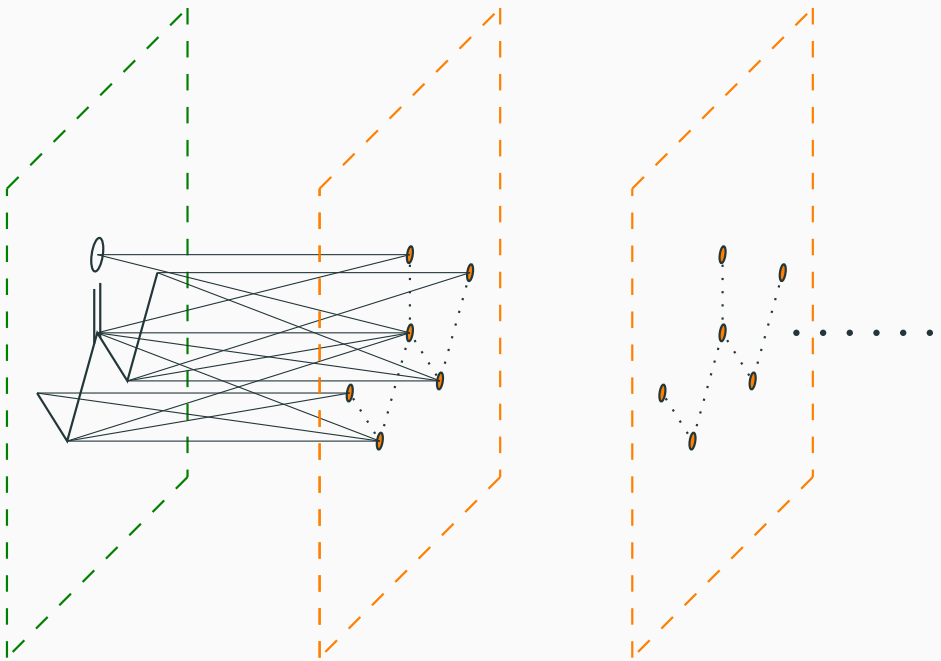

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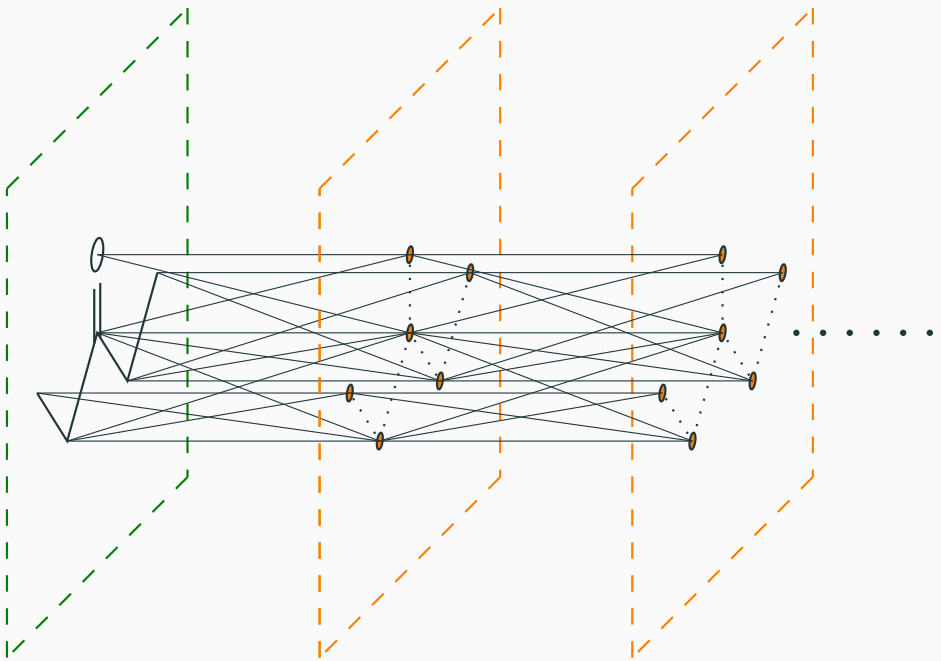

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**Goal of the project**: what can we do with graph neural networks? (from a *theoretical* perspective)

Thanks for your attention!

# Bibliography I


David K Duvenaud, Dougal Maclaurin, Jorge Iparraguirre, Rafael Bombarell, Timothy Hirzel, Alán Aspuru-Guzik, and Ryan P Adams.
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In *Advances in neural information processing systems*, pages 2224–2232, 2015.