# A Game on Directed Acyclic Graphs

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# Motivation

- Dalvi and Suciu's algorithm for computing the probability of a safe UCQ uses essentially three rules [2]:
  - Independence: $\Pr(A \wedge B) = \Pr(A) \times \Pr(B)$ when $A, B$ independent
  - Negation: $\Pr(\neg A) = 1 - \Pr(A)$
  - Inclusion-exclusion: $\Pr(A \vee B \vee C) = \Pr(A) + \Pr(B) + \Pr(C) - \Pr(A \wedge B) - \Pr(A \wedge C) - \Pr(B \wedge C) + \Pr(A \wedge B \wedge C)$

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- Knowledge Compilation-based techniques use three rules:
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An open problem in probabilistic databases: can we handle all safe UCQs using Knowledge Compilation? (via so-called deterministic decomposable circuits (d-Ds))

# The game

(Invented with Antoine Amarilli)

- We consider directed acyclic graphs (DAGs) where nodes can be "**on**" or "**off**"

→ Let $G = (V, E)$ be a DAG with all nodes being initially off

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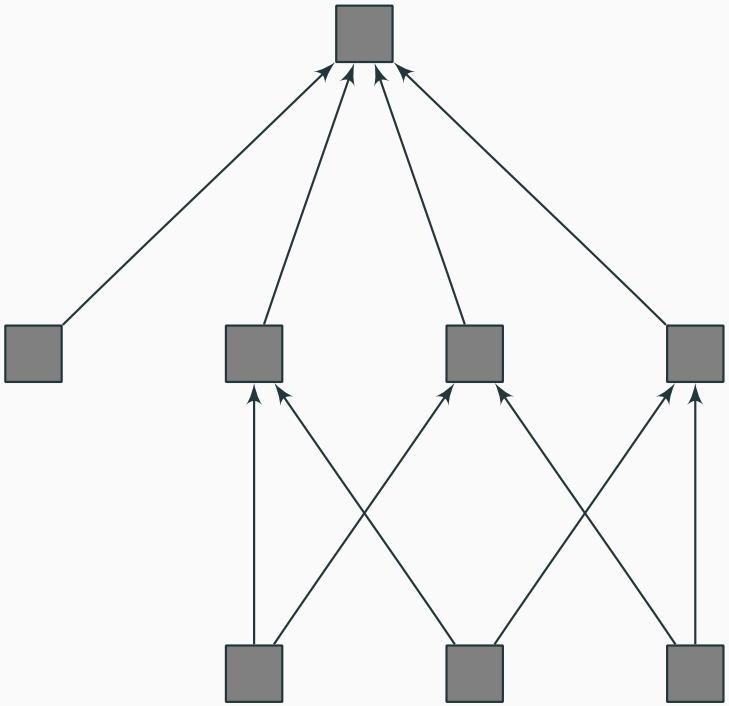
  - pick a node $u \in V$. If $u$ and all its descendants are off, then we can switch them all on; we say that we have played an on-move on $u$
  - pick a node $u \in V$. If $u$ and all its descendants are on, then we can switch them all off; we say that we have played an off-move on $u$

# Example



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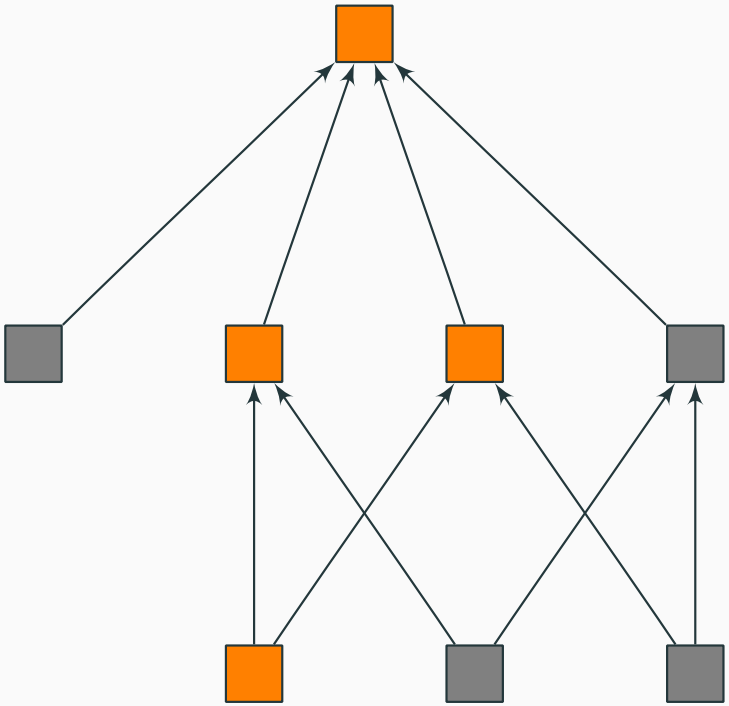

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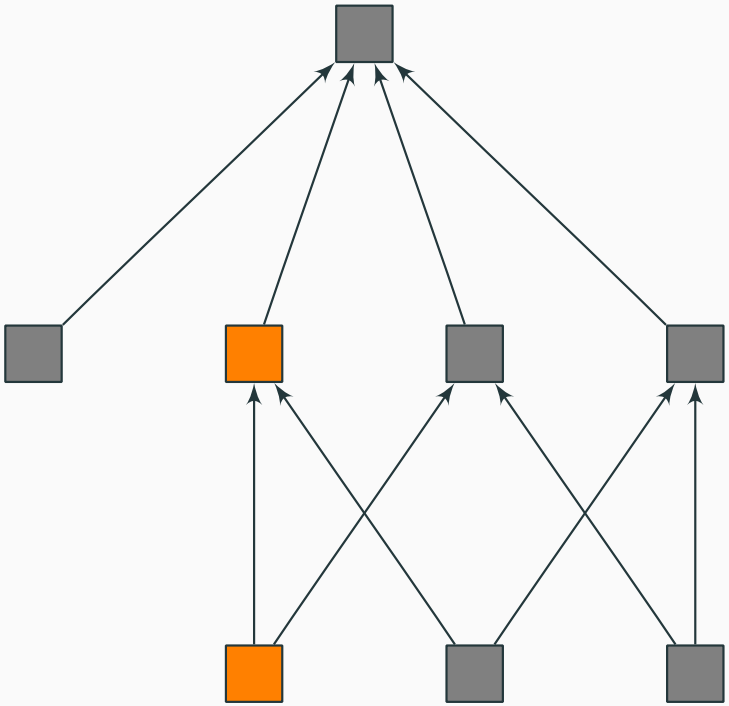

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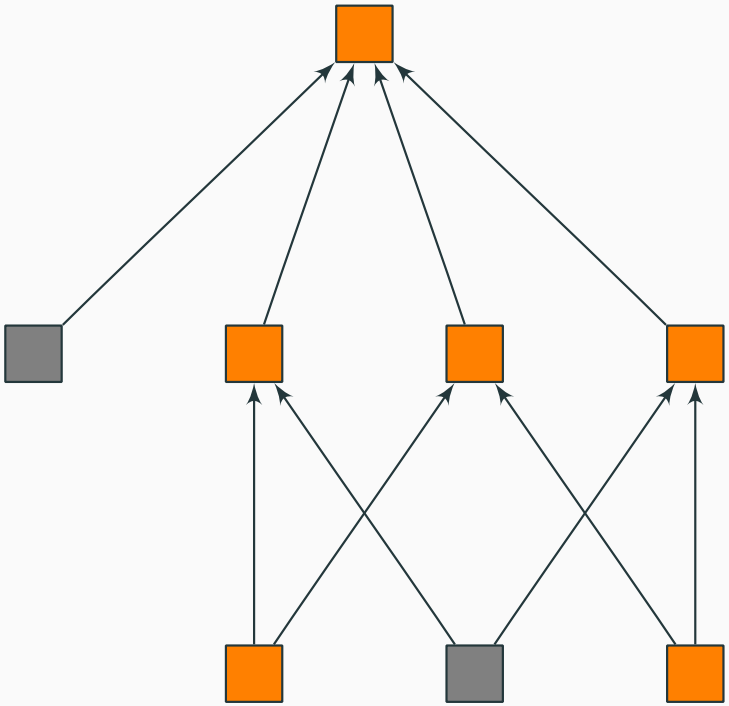

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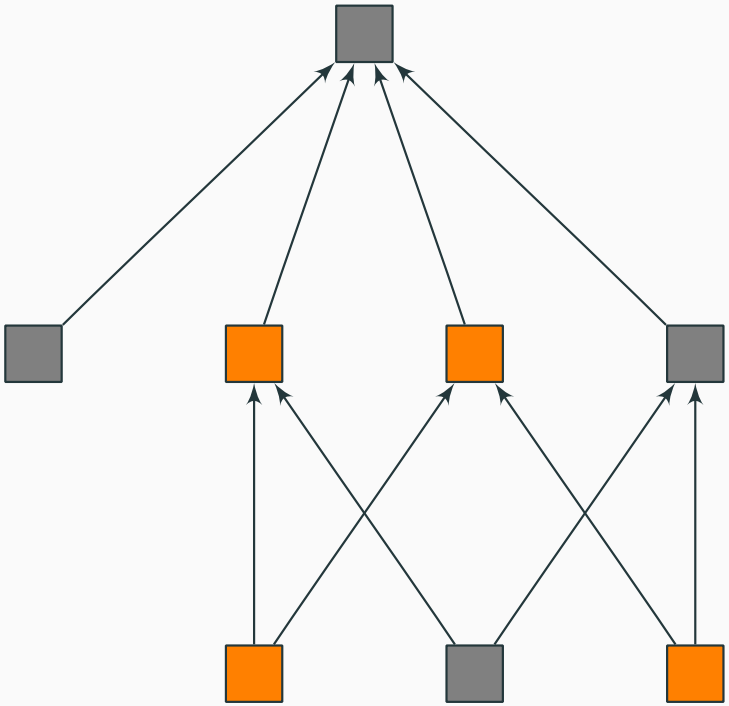

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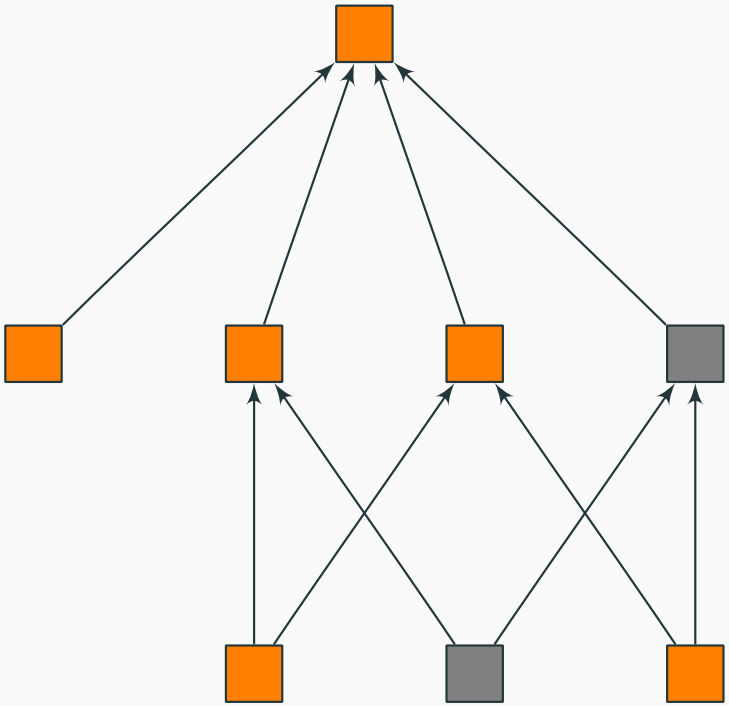

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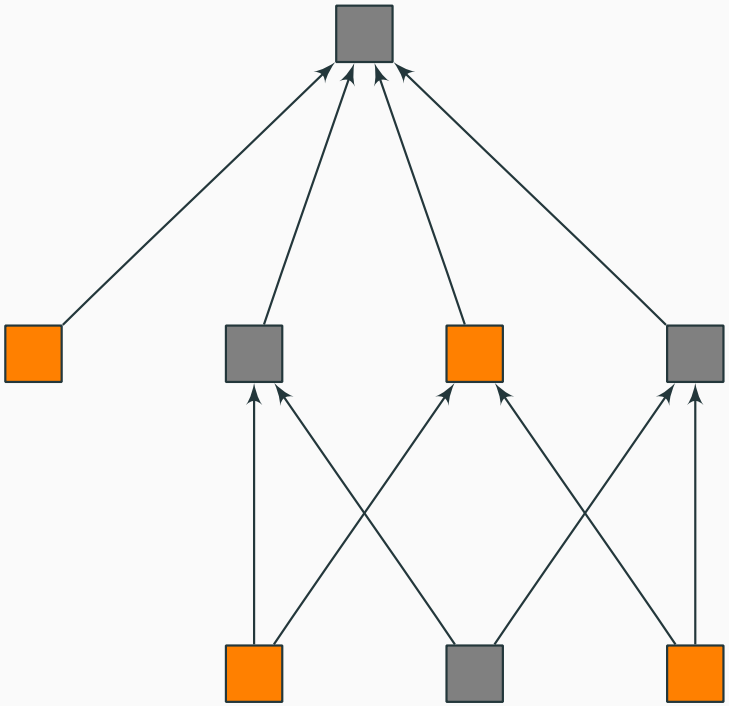

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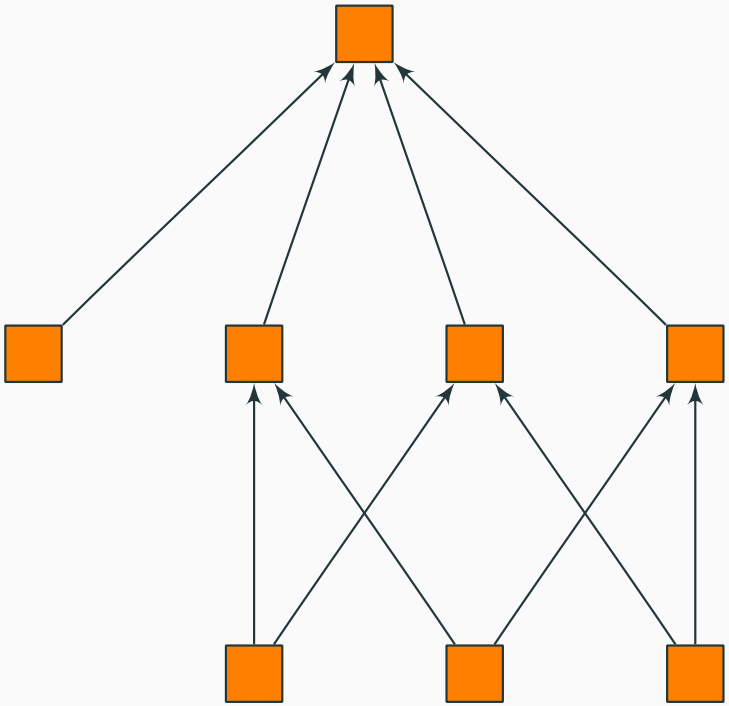

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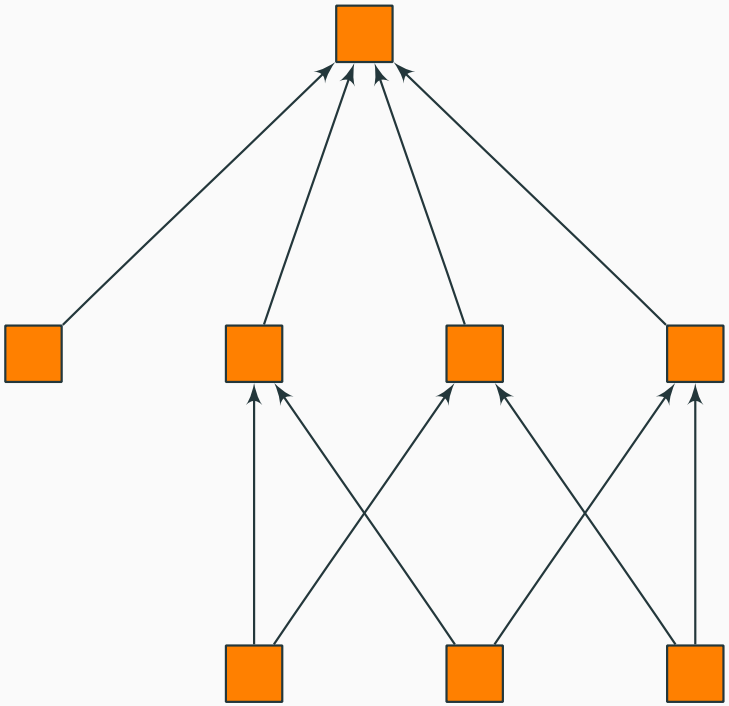

WIN!

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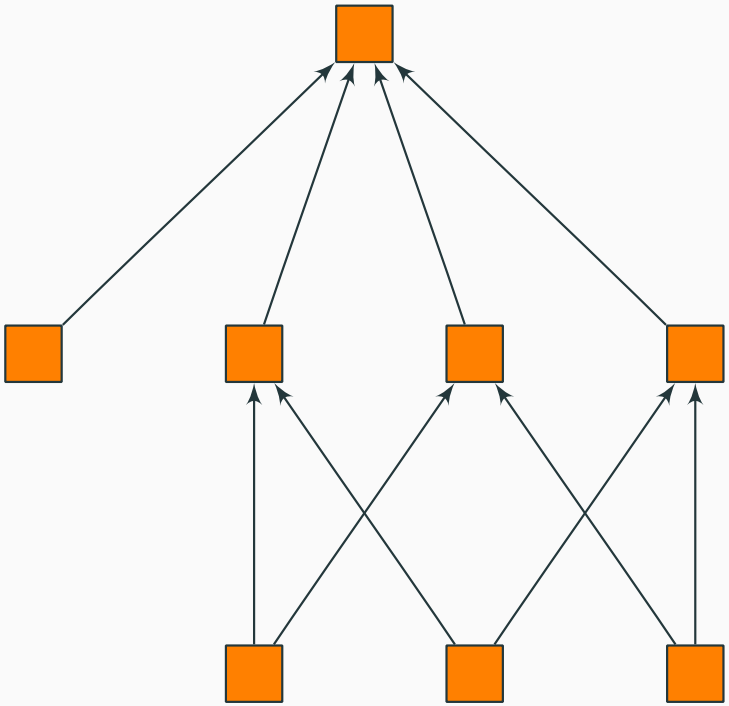

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Can be used to re-order terms in the inclusion-exclusion formula in a disjoint manner

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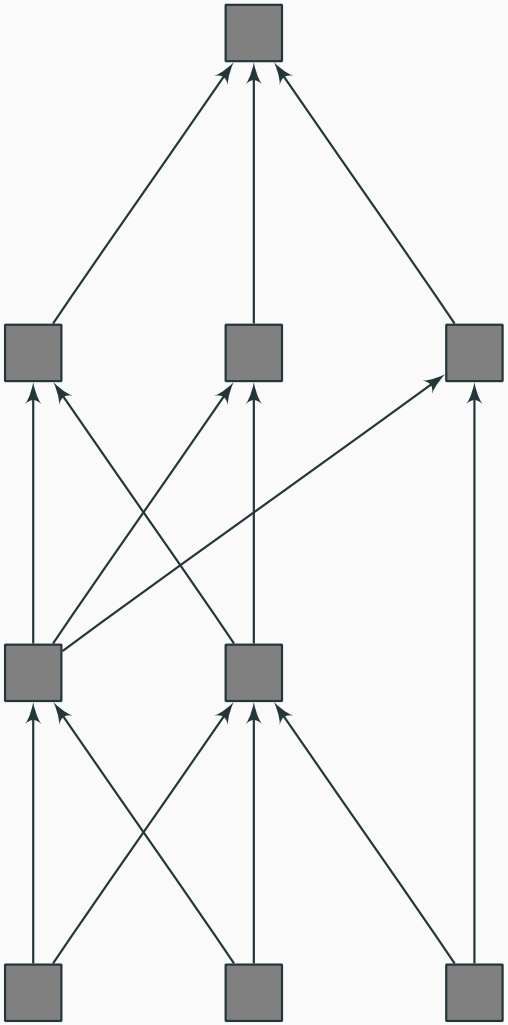
- Can we win for all the DAGs?

$\rightarrow$ No

# Counter example

Computer search by Louis Jachiet gave us this DAG



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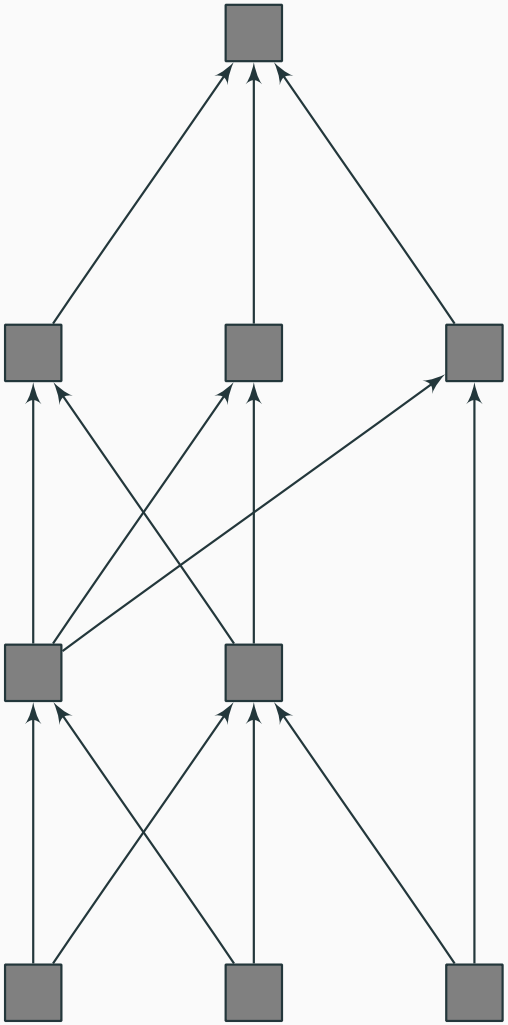

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→ Characterize the winning DAGs

→ For our purposes (constructing d-Ds for safe UCQs): Do we win when $G$ is a join semi-lattice?

- See also [1, 3]

Thanks for your attention!

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