Applications of knowledge compilation to databases counting/probabilistic problems

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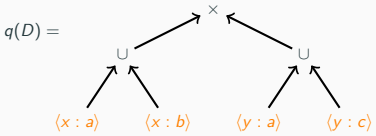
Dagstuhl seminar #26221: *Knowledge Compilation in Artificial Intelligence, Databases, and Formal Methods*

Introduction

Dan Olteanu's talk: using **multi-valued** circuits (a.k.a. *factorized databases*) to represent query results

$q(x, y) =$ a query with **free vars**

$D =$ a database



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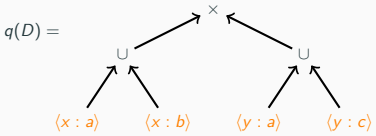
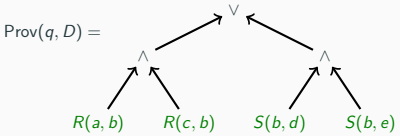


This talk: using Boolean circuits for problems that can be solved via *Boolean provenance*

q: a Boolean query
D = a database



Outline

1. Boolean provenance
2. Probabilistic databases
   - Compiling hierarchical conjunctive queries
   - Compiling MSO queries on tree-like data
3. Other problems: Shapley values, resilience
4. ProvSQL teaser

Boolean provenance

Provenance

- A (relational) database D: a finite set of *facts/tuples*
  - → **Example:** $D = \{R(a, b), R(b, c), S(b, b, a)\}$
- A Boolean query q: a function that maps databases to **true** or **false**
  - → **Example:** $q =$ "the database contains at least 10 facts"

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Definition [Imielinski & Lipski 1984]

The Boolean provenance $\mathrm{Prov}(q, D)$ is the Boolean function with facts of D as variables and such that, for any $D' \subseteq D$, $\mathrm{Prov}(q, D)$ evaluates to true on D' if and only if $D' \models q$

Boolean provenance: example

$D =$

Likes	
Alice	Bob
Alice	John
Bob	Bob
John	Bob

$q = \exists x, y, z : L(x, z) \wedge L(y, z) \wedge x \neq y$
("there are two people who like the same person")

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$$\begin{aligned}\mathrm{Prov}(q, D) = &[L(A, B) \wedge L(B, B)] \\ &\vee [L(A, B) \wedge L(J, B)] \\ &\vee [L(B, B) \wedge L(J, B)]\end{aligned}$$

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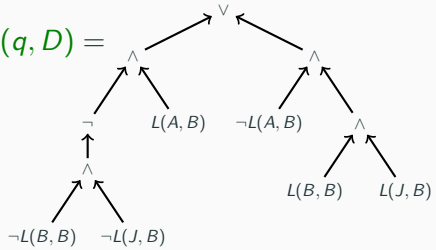

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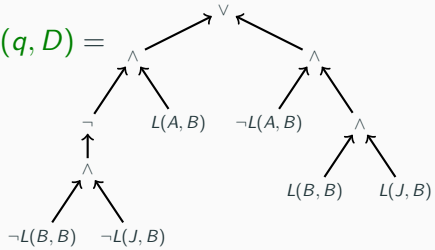

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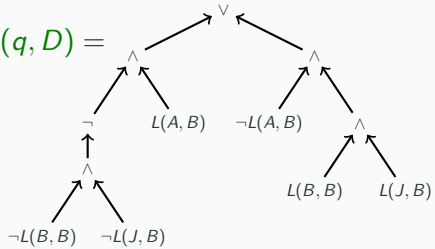

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If q is an FO sentence ($\simeq$ an SQL Boolean query), what is the data complexity of computing $\mathrm{Prov}(q, D)$? PTIME

Probabilistic databases

Tuple-independent probabilistic databases

- **Probabilistic databases**: to represent data uncertainty
  → simplest formalism: tuple-independent database

$D =$

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$\Pr(D') = (1 - 0.5) \times 1 \times (1 - 0.2) \times 0.7$

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Queries with free variables? $q(s, t) = \ldots$

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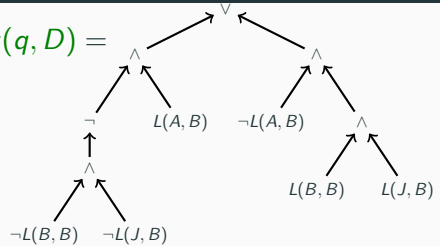

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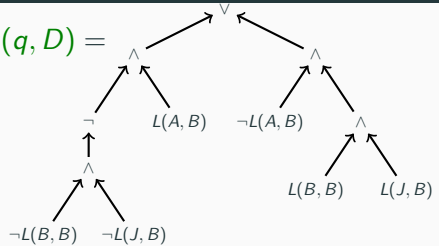

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$\Longrightarrow$ PTIME

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Definition: problem $\mathrm{PQE}(q)$, for q a Boolean query

Input: a tuple-independent probabilistic database D

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  - either $\mathrm{PQE}(q) \in \mathrm{PTIME}$, and q is called "safe"
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  - → Inclusion–exclusion:
    $\Pr(A \vee B \vee C \vee \ldots) = \Pr(A) + \Pr(B) + \ldots - \Pr(A \wedge B) - \Pr(A \wedge C) - \ldots + \Pr(A \wedge B \wedge C) + \ldots$

An open problem

- If q is a safe UCQ (i.e., $\mathrm{PQE}(q)$ is PTIME), can we compute $\mathrm{Prov}(q, D)$ as a d-D in PTIME?
  - $\rightarrow$ This is not known! (See [this arXiv note] for a reformulation as a conjecture about inclusion-exclusion)

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- But we know this can be done, for instance, when q is a hierarchical self-join free conjunctive query
  - $\rightarrow$ We prove this next

Hierarchical self-join free conjunctive query

- A conjunctive query (CQ): an existentially quantified conjunction of atoms
  $\rightarrow$ **Example:** $q = \exists x\, y : R(x, y) \wedge R(y, x) \wedge S(x, x)$

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→ Proof of this on board, using d-Ds for the tractability side ←

Monadic second-order logic on treelike data (1/2)

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- Monadic second-order logic (MSO): like FO but we have monadic (representing sets of elements) variables
  → **Example:**

$$
\begin{aligned}
q = \exists X : (&\text{Bob} \in X) \\
&\wedge (\forall y, z : y \in X \wedge \text{Likes}(y, z) \implies z \in X) \\
&\wedge \forall X' : [(\text{Bob} \in X') \\
&\qquad \wedge (\forall y, z : y \in X' \wedge \text{Likes}(y, z) \implies z \in X' \\
&\qquad \implies X \subseteq X') \\
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- Treewidth of a database: a measure indicating how close the database is to a tree

Monadic second-order logic on treelike data (2/2)

Theorem [Amarilli, Bourhis & Senellart 2016]

Let q be a Boolean MSO query, and $k \in \mathbb{N}$. Then $\mathrm{PQE}(q)$ is PTIME when restricted to databases of treewidth $\leq k$. Moreover we can build d-Ds in PTIME representing the provenance.

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$\rightarrow$ Proof (sketch) of this on board, using tree automata to build d-Ds $\leftarrow$

Other problems: Shapley values, resilience

Shapley values (1/2)

- q a Boolean query, D a database, $f \in D$ a fact of D
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  $\rightarrow$ We can use the established notion of Shapley value from cooperative game theory

Definition [Livshits, Bertossi, Kimelfeld, & Sebag 2020]

$$\text{Shapley}(q, D, f) = \sum_{S \subseteq D \setminus \{f\}} \frac{|S|!(|D| - |S| - 1)!}{|D|!} \big(q(S \cup \{f\}) - q(S)\big)$$

Shapley values (2/2)

Theorem [Deutch, Frost, Kimelfeld & Monet 2022]

- for any Boolean query q, computing Shapley values for q reduces in PTIME to $\mathrm{PQE}(q)$
- we can compute in PTIME the Shapley values of d-D circuits (bottom-up dynamic programming)

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- There is also a notion of Shapley value for probabilistic databases!
  - → See Karmakar, Monet, Senellart & Bressan 2024. In this case the problem is PTIME equivalent to PQE.

→ We can compute in PTIME (probabilistic or deterministic) Shapley values for safe queries on all database, or for MSO queries on tree-like data

Resilience

- q a Boolean query, D a database
- $\mathrm{Res}(q, D) = \min_{\substack{D' \subseteq D \\ s.t. D \setminus D' \not\models q}} |D'|$. In English: the minimum number of facts to remove from D to make the query false.

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$\rightarrow$ We can compute resilience in PTIME if we can build d-DNNFs for $\neg q$

ProvSQL teaser

ProvSQL

- ProvSQL: a PostgreSQL module to compute provenance, lead by Pierre Senellart
- Can handle other semirings than the Boolean semiring
- To solve PQE/Shapley: first compute $\mathrm{Prov}(q, D)$ as a Boolean circuit, then either
  - call a **knowledge compiler** to compile to d-DNNF
  - if the circuit's **treewidth** is small, convert it to d-DNNF using [Amarilli, Capelli, Monet & Senellart 2020]

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- To solve PQE/Shapley: first compute Prov(q, D) as a Boolean circuit, then either
  - call a **knowledge compiler** to compile to d-DNNF
  - if the circuit's **treewidth** is small, convert it to d-DNNF using [Amarilli, Capelli, Monet & Senellart 2020]

→ See **Pierre's demo on Friday**!

Thanks for your attention!

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