Weighted Counting of Matchings in Unbounded-Treewidth Graph Families

Antoine Amarilli, **Mikaël Monet**

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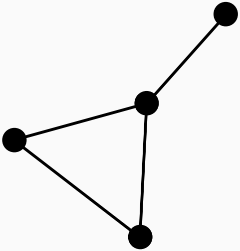
Who

Joint work with Antoine Amarilli

https://arxiv.org/abs/2205.00851

What

- Matching in a graph: set of edges that do not intersect



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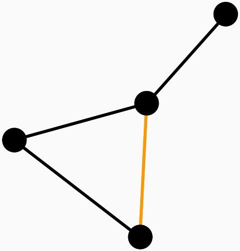

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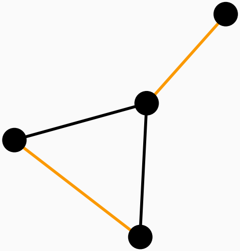

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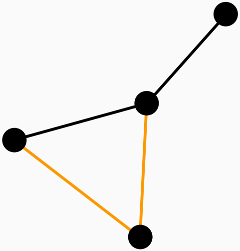

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$\Longrightarrow$ Is there another criterion than bounded treewidth that allows matchings to be counted efficiently? **No!***

* subject to defining the problem in a slighlty more general way and assuming a certain "treewidth-constructibility" requirement; see next slide for Proper Usage.

Our result

Theorem

Let $\mathcal{G}$ be an arbitrary family of graphs which has unbounded treewidth. Then the problem, given a graph $G = (V, E)$ of $\mathcal{G}$, of computing the number of matchings of G, is intractable.

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- **Counter-example**: $\mathcal{G}$ = the family of all cliques but where edges are exponentially subdivided

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Let $\mathcal{G}$ be an arbitrary family of graphs which ~~has unbounded treewidth~~ is treewidth-constructible. Then the problem, given a graph $G = (V, E)$ of $\mathcal{G}$ and rational probabilities values $\pi(e)$ for every edge of G, of computing ~~the number of matchings of G~~ the probability of a matching in G, is intractable.

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- probability of a matching in G: probability of drawing a matching when we select each edge independently with probability $\pi(e)$
- treewidth-constructible: given $k \in \mathbb{N}$ as input, we can construct in polynomial time a graph of $\mathcal{G}$ whose treewidth is $\geq k$

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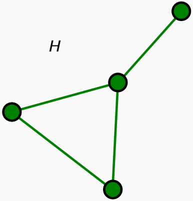

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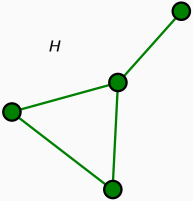
- **Step 1.** Using treewidth constructibility of $\mathcal{G}$ and the polynomial grid-minor extraction algorithm of [Chekuri and Chuzhoy, 2016], construct a graph $G \in \mathcal{G}$ such that H is a topological minor of G



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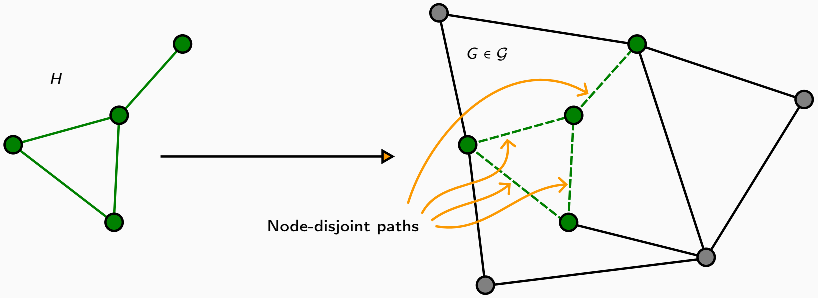

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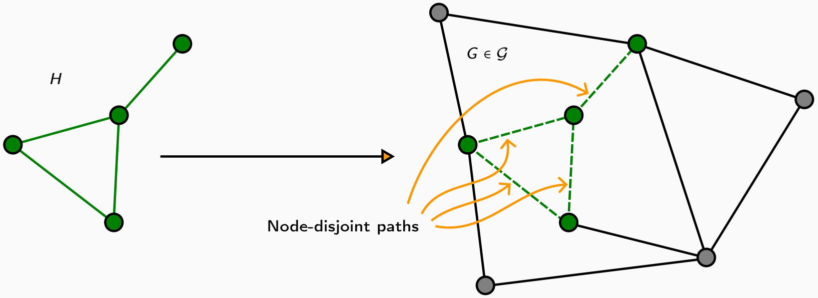

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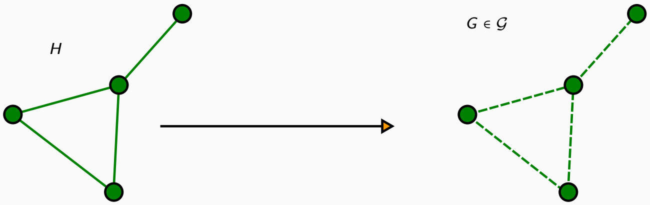
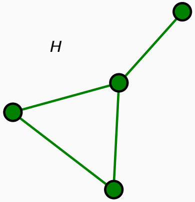
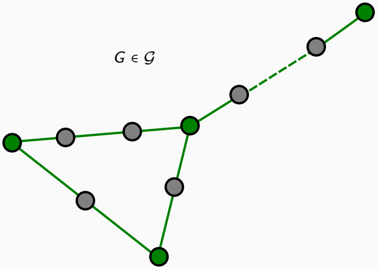


Proof sketch 2/4: How to recover the matchings?





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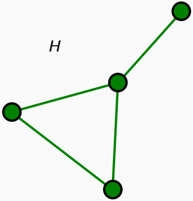

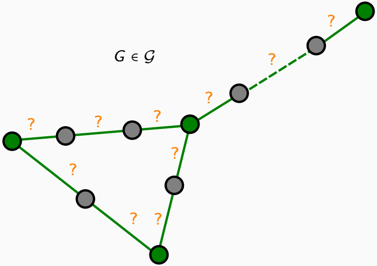

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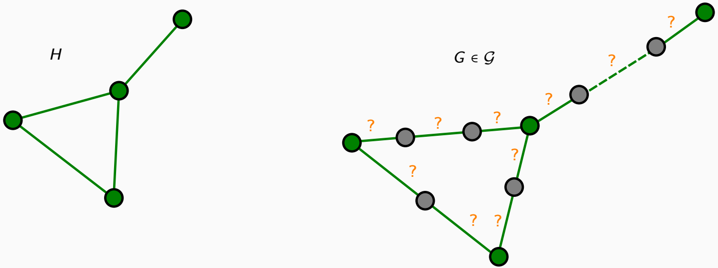
- Understand the relationship between matchings of those two graphs



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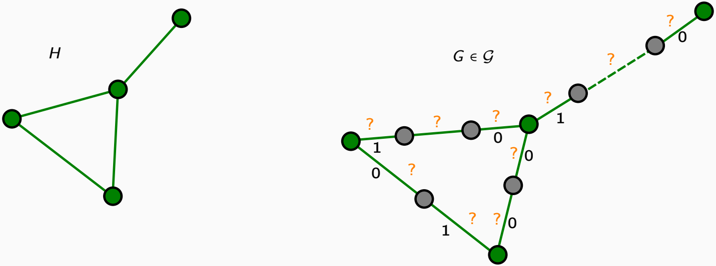

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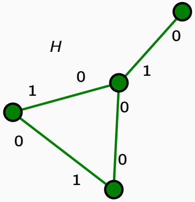

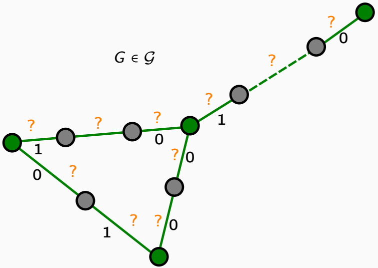

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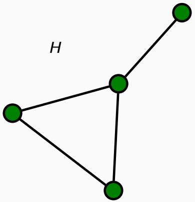
- a selection function of $H = (V, E)$ is a partial function μ that maps every vertex to at most one incident edge



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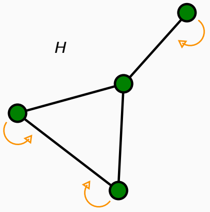

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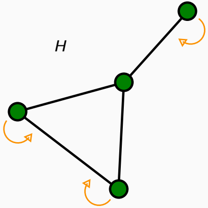

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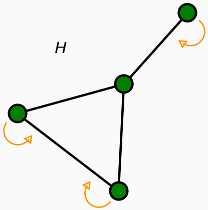

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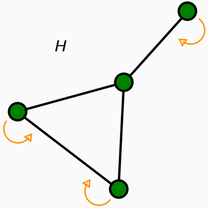

Fact

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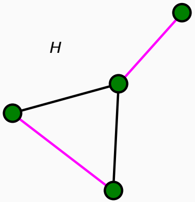

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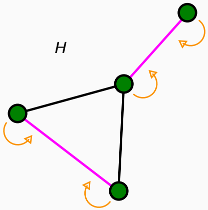

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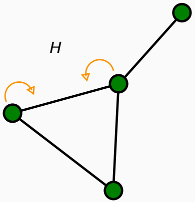

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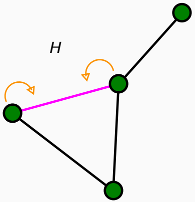

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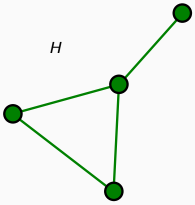

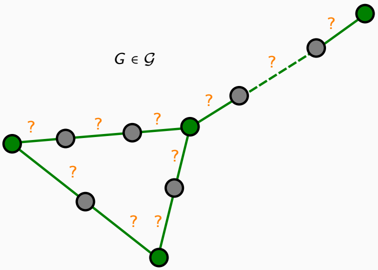

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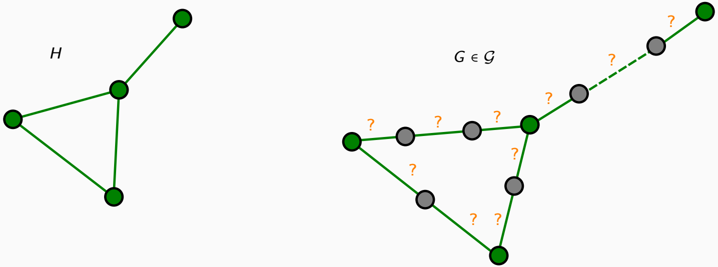

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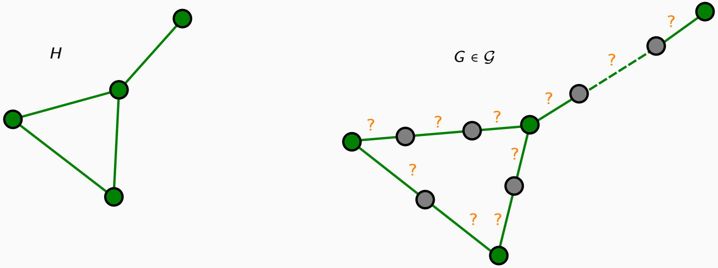

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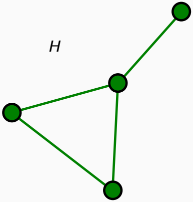

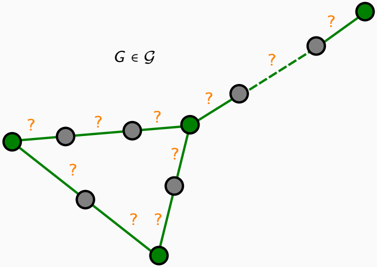

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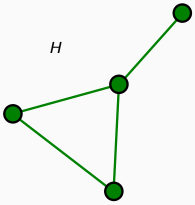

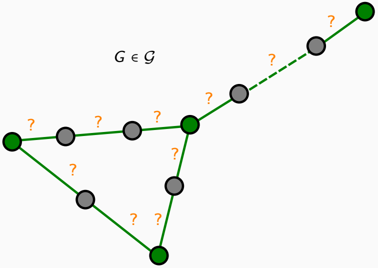

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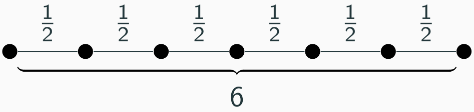

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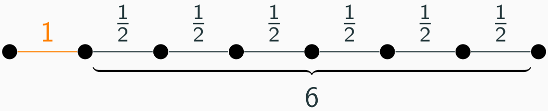

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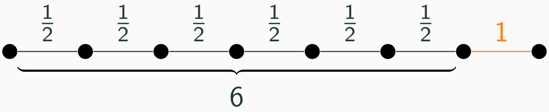

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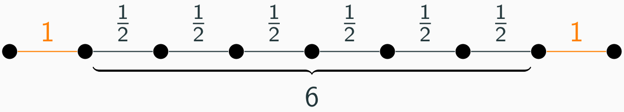

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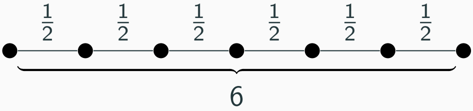

Proof sketch 4/4: A technical challenge

"Emulate" long paths with probability 1/2 with short paths:

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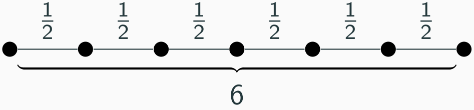

$$(p,q,r,s) = \left(\tfrac{1}{4992}\sqrt{1002921} + \tfrac{977}{1664},\ \tfrac{3}{7600}\sqrt{1002921} + \tfrac{3367}{7600},\ -\tfrac{3}{7600}\sqrt{1002921} + \tfrac{3367}{7600},\ -\tfrac{1}{4992}\sqrt{1002921} + \tfrac{977}{1664}\right)$$

Proof sketch 4/4: A technical challenge

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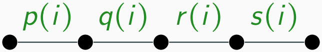

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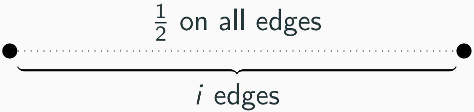

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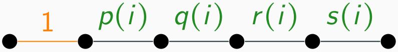

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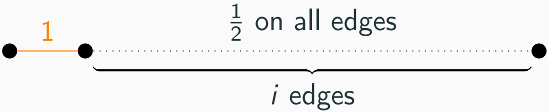

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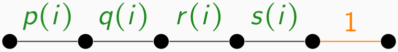

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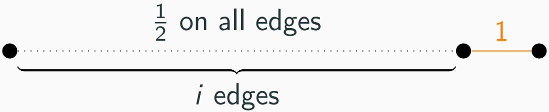

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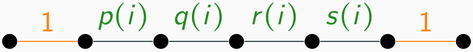

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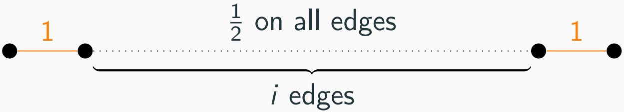

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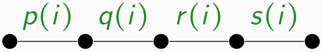

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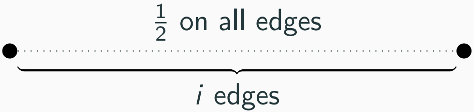

$(p(i), q(i), r(i), s(i)) = ?$

$\implies$ **This is possible when i is even and ≥ 4**

Closed form expressions for $p(i), q(i), r(i), s(i)$

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$$\begin{aligned}
C_0 &= \left(F_{-1}^4 - 2\,F_{-1}^2 + 1\right)F_{-2}^2 \\
C_1 &= 2\left(\left(F_{-1}^4 + F_{-1}^2\right)F_{-2}^3 + 2\left(F_{-1}^5 - F_{-1}^3\right)F_{-2}^2 + \left(F_{-1}^6 - 2\,F_{-1}^4 + F_{-1}^2\right)F_{-2}\right) \\
C_2 &= F_{-1}^8 + 4F_{-1}^5F_{-2}^3 + F_{-1}^4F_{-2}^4 - 2F_{-1}^6 + F_{-1}^4 \\
&\quad + 2(3F_{-1}^6 - F_{-1}^4)F_{-2}^2 + 4(F_{-1}^7 - F_{-1}^5)F_{-2} \\
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Conclusion

Theorem

Let $\mathcal{G}$ be an arbitrary family of graphs which is treewidth constructible. Then the problem, given a graph $G = (V, E)$ of $\mathcal{G}$ and rational probabilities values $\pi(e)$ for every edge of G, of computing the probability of a matching in G, is intractable.

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- also holds for edge covers (and most likely also for independent sets and vertex covers, when probabilities are on the nodes)
- but the result is false for perfect matchings! These can be counted on planar graphs by the **FKT algorithm**

Conclusion

Open: allow only probabilities in $\{0, 1/2\}$. In other words:

Open problem

Let $\mathcal{G}$ be an arbitrary family of graphs which is treewidth constructible and which is closed under taking subgraphs. Then the problem, given a graph G of $\mathcal{G}$, of computing the number of matchings in G, is intractable.

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Thanks for your attention!

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